
Homo-Hetero Pairing Regression Model: An Econometric Predictive Model of Homo Paired Data

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Abstract

The study aimed to examine the technical and fundamental hypotheses in NYSE, NASDAQ and S&P 500 stock exchange markets. The main determinants (variables) that were examined were stock trading volumes, closing stock prices and stock information available in the stock exchange market. The 240 days, 197 days and 253 days data of closing stock prices and trading volumes at NYSE, S&P500 and NASDAQ stock exchange markets were systematically collected from June 2021 to June 2022. The data was analysed by using the Homo-Hetero Pairing (HHP) Regression Model. This model was developed to detect the linear and non-linear behaviour of data. The study evidenced that both the technical and fundamental hypotheses in NYSE, S&P500 and NASDAQ stock exchange markets are defined by the inverse and S-curved models in two distinctive pairing classes called the positive-positive pairing (PPP) class and the negative-positive pairing (NPP) class. The study concluded that the optimal prediction of the stock price or return is achieved by the fundamentalists in the stock exchange markets. The study recommends that stock investors should priorities the use of the fundamental hypothesis to make their portfolio investment decision. Moreover, the study recommends the application of the HHP regression model in financial markets, economics, psychology, sociology, and medicine studies. In addition, the HHP regression model is recommended for the prediction of water waves in the investigation of hydrodynamic and erosion-accretion processes.

Keywords: Homo-Hetero Pairing Regression, Stock Price, Trading Volume, Econometric Predictive Model, Homo-Paired Data

JEL Codes: C01, C02, C18, C51, D91

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1. Introduction

The homo paired data are data of the same (homo) items sampled from the same population or level of analysis (Niroumand, Zain and Jamil, 2013). In other words, the homo paired data are

data in the couple or dyads (Teachman, Varver, and Day, 1995). The basic requirement of the homo paired data is it should be collected from two or more members /items of the same groups. In practice, these kinds of data are mostly and frequently used in our daily life activities. For example, the homo paired data may include the household data, family data, inflation rate, and economic growth sampled in a series of times (historical data). On the other hand, unpaired (hetero) data are data that sample from different populations or levels of analysis (Oyeka and Umeh, 2012). For example, the data sampled from individuals and that of the households, or data sampled from the country level (e.g., inflation rate or GDP per capita) and individual level (e.g., monthly income). These kinds of data are sampled from different levels of analysis and different populations, hence are unpaired data.

In the econometrics studies, both the short and long-run economic plans involve the prediction of the homo paired data such as inflation rate, stock prices, economic growth rate, stock trading volume, foreign direct investment (FDI) inflow and outflow in a country. These kinds of econometric data require robust predictive models. The accurate prediction of homo paired data helps the decision-makers to reduce the risk due to uncertainty planning of the economic plans. Moreover, the appropriate predicted plans use an appropriate resource and timely resourcing (managing the supply and demand). In practice, the prediction of data is done from historical data; the historical data may behave either linearly or non-linearly depending on changing factors such as time, technology, market forces, etc. The phenomena of the homo paired data behaving either linearly or non-linearly pose a predictive challenge to decision-makers in selecting which appropriate predictive model will be used (Soni, Tewari, and Krishnan, 2022). Recently, in the econometric studies, the application of artificial intelligence such as neural network analysis and the use of the learning machine techniques such a support vector machine (SVM), random forest, K-neighbour and others are mostly applied and recommended because they have shown the least predictive error (Prasad, Gumparthi, Venkataramana, ..., and Nishanthi, 2022; Srivinary, Manujakshi, Kabadi and Naik, 2022). However, the use of artificial intelligence and learning machine techniques is not supported by some studies. For example, Soni, et al. (2022) claimed that SVM is mostly demonstrating high accuracy on non-linear classification data; the random forest method demonstrates high accuracy in the binary classification data. In other words, some of the artificial and learning machine methods can distort the predictive accuracy if they are used inappropriately. Another drawback is that artificial intelligence method such as neural network analysis, it not interpreted directly and deep neural analysis require more training time and dimensional reduction of the data (Seethalakshmi, 2018; Enke, Grauer and Mehdiyev, 2011; Guraray, Shriya and Ashwini, 2019). Moreover, Prasad, et al. (2022) contended the application of the learning machine and artificial intelligence methods does not grant predictive accuracy, in comparison to the statistical methods.

One of the active econometric challenges is the prediction of the stock price in the stock exchange market (Wang, 2014; Srivinary, et al. 2022; Prasad, et al. 2022; Gharehchopogh, Bonab and Khaze, 2013; Bhuriya, Kaushal, Sharma, and Singh, 2017). The stock prices are one of the homo paired data that are volatile due to the different factors that influence the stock price at the stock exchange market. Traditionally, the prediction of the stock price was done by technical aids (price and trading volume trends) and brokers. This prediction was based on historical prices, volume, and price trends (Sahoo and Charlapally, 2015). Today stock prediction

has become very complex than before as stock prices are not only affected by the company's related information but also socio-economic conditions of the country, political atmosphere and others (Sahoo and Charlapally, 2015). Specifically, the stock prices are influenced by internal factors such as the company's profitability, liquidity, asset tangibility, management or governance effectiveness of the company, and goodwill. On the other hand, the stock prices are influenced by external factors such as the inflation rate, level of economic growth, government policy, global market uncertainty, the interest rate of the financial markets, etc (Bhuriya, et al., 2017). Notably, the returns from the stock market are always uncertain and ambiguous hence traditional techniques will not give accurate predictions (Sahoo and Charlapally, 2015). In most practice, the homo paired data in a short term behave linearly but in the long run, behave non-linearly. In the stock market prices, the data may behave either linearly or non-linearly in the short or long term, this phenomenon increases the complexity of the prediction of the stock price at the stock exchange market. Adversely, the inaccuracy predictive model increases the risk of trading on stocks, hence hurting economic growth. A lot of studies have been made in this area and advanced intelligence techniques ranging from the pure mathematical model and expert system to neural networks have been proposed for stock predictive, however, they have been evidenced to have methodological drawbacks (Sahoo and Charlapally, 2015; Singh, Rehan and Kumar, 2022; Demiray and Gurhanli, 2021).

Some studies such as Karin, Alam and Hossain (2021) use linear regression and decision tree regression to analyse the stock price and conclude that the stock market is a more perplexing and sophisticated way of doing business. Gururay, et al. (2019) showed that linear regression is the basic tool by which linear trend is obtained. However, Gharehchopogh, et al.(2013) evidenced that the market fluctuation affects the stock price and trading volume which became hard to predict, therefore increasing the risk of the investors' returns. That is, a better predictive model is required to assist the investors to trade the stock with greater certainty. That is, the investors do not know when the price will go up (increase) or down (decrease). Gharehchopogh, et al.(2013) suggested that data mining techniques have more successful performance in predicting various fields such as economy, policy and engineering compared to traditional statistical methods by discovering the hidden knowledge of the data. Moreover, the experience has shown that learning machine techniques can be successfully predicting daily stock prices and their trading volume. In that sense, Gharehchopogh, et al.(2013) contradicts Prasad, et al. (2022), Sahoo and Charlapally (2015) and Karim, et al.(2021) who advocate for statistical methods. Bhuriya, et al.(2017) stated that the stock market is a messy spot for anticipating since there are not any critical guidelines (model) to access or foresee the estimation of the offer(price) inside the stock market. They contended that numeral techniques like specialised investigation, principal examination, factual examination and so forth arrange to anticipate the value inside the stock market nevertheless none of these approaches is incontestable as a faithfully worthy expectation instrument. They conclude that stock price data is notoriously difficult or impossible to predict. The linear regression fails when a randomly or stochastic and unpredictable event occurs. That is, not every stock price has a steady linear change or trend. Moreover, Bajzik (2020) concluded that one can not rely on any general conclusions about stock markets; the predictability of the stock returns varies with different markets and stock types. Enke, et al.(2011) suggested that to have a predictive model that predicts stock price level precisely, the model should consider or capture

the non-linearity and discontinuities of the factors which are considered to impact the stock market.

From the literature, we notice that there is no consensus on which predictive model is the best or optimal for stock price prediction. Some studies concluded that the prediction of the stock price in the stock exchange market is impossible (Bhuriya, 2017). Moreover, some studies advocate for artificial intelligence and learning machine techniques (Seethalakshmi, 2018; Smith and Rajan, 2017; Cai and Gao, 2017; Wang, 2014; Srivinary et al., 2022; Soni et al., 2022). On the other hand, some studies advocate for traditional statistical methods (Prasad et al., 2022; Chaudhary, Arora and Singh, 2018; Gupta and Wagalakshmi, 2019; Mali, Karchalkar, Jain,..and Kumar, 2017). Therefore, the researchers reached the opposite diametric conclusion on the best choice of the powerful predictive model of the stock price. The literature exhibits the contradictory evidence gaps that this paper forced to establish a new statistical model that predicts the data in pairs.

On the other hand, the study on the relationship between the stock price and trading volume is still debatable in the literature. Mahajan and Singh (2008) found in the Indian Stock Market that there is a weak causality running from volume to return (price). That is, volume is an informational signal to investors. Mpofu (2012) found a positive relationship between the stock price and contemporary changes in trading volume in the JSE security exchange in South Africa. This study was supported by Pathirawasam(2011) on the Colombia stock exchange. Kaizoji (n.d) studying the Tokyo stock exchange found that the relationship between the stock price and trading volume is defined by power laws. A recent study by Alhussayen (2022) in the Saudi Arabia stock market evidenced that there is a unidirectional relationship running from a return to volume. He concluded that the volume does not carry informational content and cannot predict the stock price. That is, the returns (price) do impact the trading volume but the effect is not steady, which also causes the problem of prediction. This conclusion contradicts Mahajan and Singh (2008). Karpoff (1987) confirmed that volume is positively related to the magnitude of the price change which also supports Mahajan and Singh (2008) and contradicts Alhussayen (2022). The specific studies on NYSE by Huang and Heian(2010) using the conventional method developed by Jegadeesh and Titman (1993) found there is a statistically abnormal return for higher volume. However, the historical studies in NYSE by Granger and Morgenstern (1963) using the same approach found that a high trading volume tends to follow high returns and vice versa. The recent study on NYSE done by Yoni (2013) found that there is a positive contemporaneous relationship between return and trading volume, which contradicts Huang and Heian(2010) and supports Granger and Morgenstern (1963) and Gallant and Rossi (1990). The fundamentalists' studies such as Huana, Capretz and Ho (2021), Malinowska (2010), Wu, Liu, Zou and Weng(2022) and Bajzik (2020) empirically evidence that the stock price adjustment is due to the information released. That is, the price response to public new information is gradual and accompanied by increases in trading activities.

Specifically, the study examined the impact of homo pairing changes in daily stock information relatively to changes in daily closing stock price on homo pairing changes in daily closing stock prices relative to changes in daily stock information (fundamental analysis), and the impact of homo pairing changes in the daily closing stock price relative daily trading volume on homo

pairing changes in daily trading volume relative to daily closing stock (Technical analysis) in New York Stock Exchange Market (NYSE), Standard and Poor's 500 (S&P 500) and the National Association of Securities Dealers Automatic Quotation System (NASDAQ). In other words, the study asks “does daily stock information available on the stock exchange market significantly influence the next (future) price level of the stock?” On the other hand, the study examined if the statistical data (records) of daily stock prices and their trading volume convey a significant message or signal to the investor’s decisions. The next sections of the paper cover the concept and model development, methodology, findings, conclusion and recommendation, and references.

2. Literature Review

2.1. The Concept and Model Development

The homo-paired data may behave linearly or non-linearly; data moves up and down for a range of periods. The data that may acquire both behaviours of linearity and non-linearity, its prediction will not be accurately done by the traditional linear regression (Bhuriya et al., 2017; Gharehchopogh, 2013; Karin et al., 2021; Gururay et al., 2019). It requires a model that can capture the up and down movement of data; that is the non-linearity and discontinuities of the data (Enke, et al., 2011). The study used the homo-hetero pairing (HHP) regression model. The HHP regression model tackled the question of how the homo pairing changes in x relative to change in y can accurately predict the homo pairing changes in y relative to changes in x . The term “pairing changes” means the “*net pairing change effect*” of the two paired variables x and y , that changes from one value to the next value, that is, x_1 to x_2 and y_1 to y_2 . Therefore, the net pairing change effect is $(x_2 - x_1) - (y_2 - y_1)$. This can be expressed as the percentage of change in x , $(x_2 - x_1)$ and become x -homo pairing changes, ∂_x , i.e., $[(x_2 - x_1) - (y_2 - y_1) / (x_2 - x_1)]$. When the net pairing change effect is divided by change in y , $(y_2 - y_1)$ that $[(x_2 - x_1) - (y_2 - y_1) / (x_2 - x_1)]$, the equation becomes “ y -homo pairing changes, ∂_y ”.

For a clear illustration of the concept of the homo-hetero pairing regression model, let's consider the difference between the traditional linear regression modelling approach and the homo-hetero pairing regression modelling. In the traditional linear regression modelling the estimated or predicted value $\hat{y}_n$ is due to the new value of x_n with no memory of the past data, x_{n-1} . That is $x_1 \rightarrow \hat{y}_1 \dots x_n \rightarrow \hat{y}_n$. This means that x_{n-1} is useless in the estimation or prediction of $\hat{y}_n$. On the other hand, in the homo-hetero pairing regression model the estimated or predicted y -homo pairing changes relative to changes in x , (∂_y) or simply relative x -homo pairing changes in y are due to the x -homo pairing changes relative to y , (∂_x) or relative y -homo pairing change in x . That is, x_1 and y_1 are involved in the estimation of y_2 (Table 1).

Table 1: The traditional linear and homo-hetero pairing regression modelling

Traditional Linear Regression Model			Homo-hetero Pairing Regression Model		
X	Y	Description	X	Y	Description
x_1	y_1	$\hat{y}_n$ is due to the new value of x_n with no memory of x_{n-1} . That is $x_1 \rightarrow \hat{y}_1 \dots x_n \rightarrow \hat{y}_n$. That is x_{n-1} is useless in the estimation of $\hat{y}_n$.	∂_{x_1}	∂_{y_1}	$\hat{\partial}_y$ is due to the homo pairing change effect/difference of x_n and x_{n-1} . That is, x_1 and y_1 are involved in the estimation of y_2
x_2	y_2		∂_{x_2}	∂_{y_2}	
x_3	y_3		∂_{x_3}	∂_{y_3}	
.	.		.	.	
.	.		.	.	
.	.		.	.	
x_n	y_n		∂_{x_n}	∂_{y_n}	
Modelling: Regression: $X \rightarrow Y = x_n \rightarrow \hat{y}_n$ Correlation: $X \leftrightarrow Y = x_n \leftrightarrow y_n$				Modelling: Regression: $\partial_x \rightarrow \hat{\partial}_y$ Correlation: $\partial_x \leftrightarrow \partial_y$	

Source: Author (2022).

Where,

$$\partial_x = \frac{(x_n - x_{n-1}) - (y_n - y_{n-1})}{x_n - x_{n-1}}$$

$$\partial_y = \frac{(y_n - y_{n-1}) - (x_n - x_{n-1})}{y_n - y_{n-1}}$$

$$\partial_x \rightarrow \hat{\partial}_y = \left[\frac{(x_n - x_{n-1}) - (y_n - y_{n-1})}{x_n - x_{n-1}} \right] \rightarrow \left[\frac{(y_n - y_{n-1}) - (x_n - x_{n-1})}{y_n - y_{n-1}} \right]$$

The predictions of the y -homo pairing changes relative to the changes in x can be done by establishing the linear regression between the y -homo pairing changes relative to changes in x (dependent) and x -homo pairing changes relative to changes in y (independent); hereafter, y -homo pairing changes and x -homo pairing changes respectively. The common principle of least squares can be applied to establish the “homo-hetero pairing regression model”; the model that predicts the y -homo pairing changes (dependent). That is the regression modelling of the x -homo pairing changes on y -homo pairing changes. Now, let us predict the y -homo pairing changes by a homo-hetero pairing (HHP) regression model, that is,

$$\partial_y = \lambda_0 + \sum_{i=1}^n \lambda_i \partial_{x_i} + \varepsilon_i$$

$$\varepsilon_i = \partial_{y_i} - \hat{\partial}_{y_i}$$

If the HHP regression model is reduced to the HHP simple regression model, then the equation becomes,

$$\partial_y = \lambda_0 + \lambda_1 \partial_{x_i} + \varepsilon_i$$

The term error or residuals of the model, ε is determining the prediction accuracy of the model. It is measured as the difference between the actual value of the y-homo pairing changes (∂_{y_i}) and predicted y-homo pairing changes ($\hat{\partial}_{y_i}$), that is, $\varepsilon_i = \partial_{y_i} - \hat{\partial}_{y_i}$. This value can be illustrated by using the graph (Figure 1).

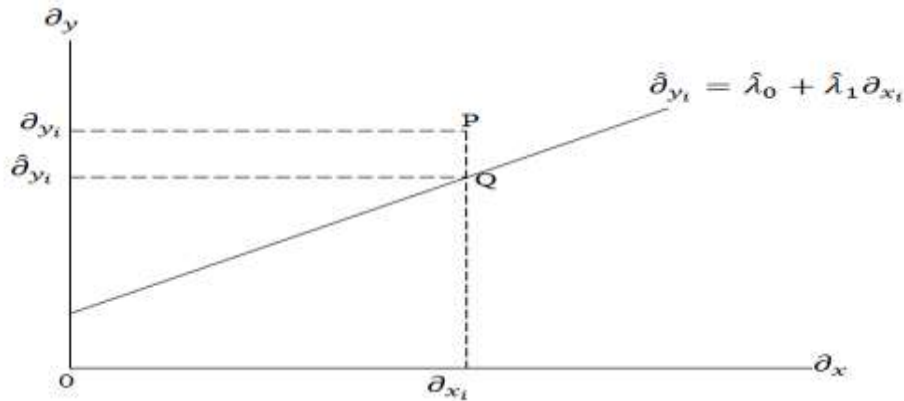


Figure 1: The profiling of residuals of the homo-hetero pairing regression model

The residuals of the model are given $\varepsilon_i = \partial_{y_i} - \hat{\partial}_{y_i}$, that is determined by the differences between the values P and Q, that is, P-Q, which measures the vertical distance of the y-homo pairing changes, (∂_y). The coordinate or data point at Q is ($\partial_{x_i}, \hat{\partial}_{y_i}$) and the data point at point P is ($\partial_{x_i}, \partial_{y_i}$). The difference between the actual value of y-homo pairing changes, (∂_{y_i}) at point P and the predicted value of the y-homo changes, ($\hat{\partial}_{y_i}$) at point Q is called the error or residual. It is the vertical distance of the y-homo pairing changes, (∂_y). It can be obtained by P-Q, that is, ($\partial_{x_i}, \partial_{y_i}$) - ($\partial_{x_i}, \hat{\partial}_{y_i}$) which is equal to ($\partial_{y_i} - \hat{\partial}_{y_i}$) for a single point data, ∂_{x_i} . For data set ∂_x with a subset $\partial_{x_1}, \partial_{x_2} \dots \partial_{x_n}$. The total residuals are given by the summation of ε_i , that is $\sum_{i=1}^n (\partial_{y_i} - \hat{\partial}_{y_i})$. The principle of the HHP equation is based on the minimisation of the sum of squared residuals or errors (SSR or SSE). The SSE is given by,

$$SSE = \sum_{i=1}^n \varepsilon_i^2 = \sum_{i=1}^n (\partial_{y_i} - \hat{\partial}_{y_i})^2$$

$$\text{since } \hat{\partial}_{y_i} = \hat{\lambda}_0 + \hat{\lambda}_1 \partial_{x_i}$$

$$SSE = \sum_{i=1}^n \varepsilon_i^2 = \sum_{i=1}^n \left(\partial_{y_i} - \hat{\lambda}_0 + \sum_{i=1}^n \hat{\lambda}_i \partial_{x_i} \right)^2, \text{ HHP multiple regression Model}$$

$$SSE = \sum_{i=1}^n \varepsilon_i^2 = \sum_{i=1}^n (\partial_{y_i} - \hat{\lambda}_0 - \hat{\lambda}_1 \partial_{x_i})^2, \text{ HHP simple regression model}$$

Since the optimal parameter $\hat{\lambda}_0$ and $\hat{\lambda}_1$ will be obtained at the minimal point of the function curve of SSE. Therefore, we differentiate the function of SSE to determine where its slope is equal to zero. That is the point value that there is no increase of error or residual. This means the point that the estimated value or predicted value is close to the actual values of the paired data. Taking the SSE function of the HHP simple regression model, we find its first derivative with respect to their parameters, $\hat{\lambda}_0$ and $\hat{\lambda}_1$. That is,

$$\frac{d(\sum_i \varepsilon_i^2)}{d\hat{\lambda}_0} = \frac{d \sum_i (\partial_{y_i} - \hat{\lambda}_0 - \hat{\lambda}_1 \partial_{x_i})^2}{d\hat{\lambda}_0} = 2 \cdot (-1) \cdot \sum_i (\partial_{y_i} - \hat{\lambda}_0 - \hat{\lambda}_1 \partial_{x_i})$$

$$\frac{d(\sum_i \varepsilon_i^2)}{d\hat{\lambda}_1} = \frac{d \sum_i (\partial_{y_i} - \hat{\lambda}_0 - \hat{\lambda}_1 \partial_{x_i})^2}{d\hat{\lambda}_1} = 2 \cdot (-\partial_{x_i}) \cdot \sum_i (\partial_{y_i} - \hat{\lambda}_0 - \hat{\lambda}_1 \partial_{x_i})$$

These equations (slopes of the SSE function curves) are commonly known as the slopes (normal) equations which can be jointly solved to obtain the algebraic equations of the parameters $\hat{\lambda}_0$ and $\hat{\lambda}_1$. Now, since the first derivative of the SSE function is the slope or rate of error term which changes as the independent variables change, then to have an optimal linear model, the error rate should be equal to zero in the estimate in $\hat{\lambda}_0$ and $\hat{\lambda}_1$, therefore, slope equations are equalized to zero,

$$2 \cdot (-1) \cdot \sum_i (\partial_{y_i} - \hat{\lambda}_0 - \hat{\lambda}_1 \partial_{x_i}) = 0$$

$$\sum_i \partial_{y_i} - n\hat{\lambda}_0 - \hat{\lambda}_1 \sum_i \partial_{x_i} = 0$$

$$\hat{\lambda}_0 = \frac{1}{n} \sum_i \partial_{y_i} - \hat{\lambda}_1 \frac{1}{n} \sum_i \partial_{x_i}$$

$$\hat{\lambda}_0 = \bar{\partial}_{y_i} - \hat{\lambda}_1 \bar{\partial}_{x_i}$$

$$2 \cdot (-\partial_{x_i}) \cdot \sum_i (\partial_{y_i} - \hat{\lambda}_0 - \hat{\lambda}_1 \partial_{x_i}) = 0$$

$$\sum_i \partial_{x_i} \partial_{y_i} - \hat{\lambda}_0 \sum_i \partial_{x_i} - \hat{\lambda}_1 \sum_i \partial_{x_i}^2 = 0$$

substitute $\hat{\lambda}_0 = \bar{\partial}_{y_i} - \hat{\lambda}_1 \bar{\partial}_{x_i}$

$$\sum_i \partial_{x_i} \partial_{y_i} - (\bar{\partial}_{y_i} - \hat{\lambda}_1 \bar{\partial}_{x_i}) \sum_i \partial_{x_i} - \hat{\lambda}_1 \sum_i \partial_{x_i}^2 = 0$$

$$\sum_i \partial_{x_i} \partial_{y_i} - \bar{\partial}_{y_i} \sum_i \partial_{x_i} - \hat{\lambda}_1 \left(\sum_i \partial_{x_i}^2 - \bar{\partial}_{x_i} \sum_i \partial_{x_i} \right) = 0$$

$$\sum_i \partial_{x_i} \partial_{y_i} - \bar{\partial}_{y_i} \sum_i \partial_{x_i} = \hat{\lambda}_1 \left(\sum_i \partial_{x_i}^2 - \bar{\partial}_{x_i} \sum_i \partial_{x_i} \right)$$

$$\hat{\lambda}_1 = \frac{\sum \partial_{x_i} \partial_{y_i} - \bar{\partial}_{y_i} \sum \partial_{x_i}}{(\sum \partial_{x_i}^2 - \bar{\partial}_{x_i} \sum \partial_{x_i})}$$

Therefore, we get the normal equations of the least squared errors or residuals, which are

$$\begin{aligned}\hat{\lambda}_1 &= \frac{\sum \partial_{x_i} \partial_{y_i} - \bar{\partial}_{y_i} \sum \partial_{x_i}}{(\sum \partial_{x_i}^2 - \bar{\partial}_{x_i} \sum \partial_{x_i})} \\ \hat{\lambda}_0 &= \bar{\partial}_{y_i} - \hat{\lambda}_1 \bar{\partial}_{x_i}\end{aligned}$$

Now, we can recall the covariance term of ∂_{x_i} and ∂_{y_i} given by,

$$\sigma_{\partial_{x_i} \partial_{y_i}} = \frac{1}{n} \sum (\partial_{x_i} - \bar{\partial}_x)(\partial_{y_i} - \bar{\partial}_y)$$

And the variance of ∂_{x_i} is given by

$$\partial_{x_i}^2 = \frac{1}{n} \sum (\partial_{x_i} - \bar{\partial}_x)^2$$

Therefore, $\hat{\lambda}_1$ can be simplified to,

$$\hat{\lambda}_1 = \frac{\sigma_{\partial_{x_i} \partial_{y_i}}}{\partial_{x_i}^2}$$

2.2.1. Goodness-of-fit statistics

The goodness-of-fit statistics are statistics that explained the predictive accuracy of the linear model. In other words, goodness-of-fit statistics are statistics that measure the degree of the deviation between the predicted and observed values of the y-homo paired changes. That is explained by the difference between the observed value and the predicted value of the y-homo pairing changes. The common goodness-of-fit statistics of the HHP regression are y-homo paired Ψ^2 , y-homo paired adjusted Ψ^2 , homo-hetero paired P-values, y-homo paired root MSE, and homo-hetero paired t-value, homo-hetero paired confidence interval and y-homo paired F-statistic value.

2.2.1.1. The y-homo paired Ψ^2

The y-homo paired Ψ^2 , or simply Ψ^2 is the homo-hetero pairing coefficient that determines the line of best fit of the HHP regression model. It is the common measure of goodness-of-fit of the HHP regression model. It is given as,

$$\Psi^2 = 1 - \frac{SSE}{SST} \text{ or } \frac{SSR}{SST}$$

Where SSE (sum of squared residuals) = $\sum_{i=1}^n (\partial_{y_i} - \hat{\partial}_{y_i})^2$, SST (total sum of squares) = $\sum_{i=1}^n (\partial_{y_i} - \bar{\partial}_y)^2$ and SSR (regression sum of squares) = $\sum_{i=1}^n (\hat{\partial}_{y_i} - \bar{\partial}_y)^2$. Alternatively, SSR can be obtained by the formula, $SSR = SST - SSE$. Therefore, Ψ -Squared can be obtained by the formula,

$$\Psi^2 = \frac{SSR}{SST} = \frac{\sum_{i=1}^n (\hat{\partial}_{y_i} - \bar{\partial}_y)^2}{\sum_{i=1}^n (\partial_{y_i} - \bar{\partial}_y)^2}$$

The value of Ψ^2 is ranging from 0 to 1, that is $0 \leq \Psi^2 \leq 1$. The value of Ψ^2 near or close to one indicates the data is well paired in the model. That is, the regression sum of squares (SSR) is closely equal to the total sum of squares (SST). On the other hand, the value of Ψ^2 near or close to zero indicates that the model is poorly paired in the model. If Ψ^2 is equal to 1, indicates the perfect pairing of the data in the model, and if Ψ^2 is equal to 0 indicates the model is perfectly poorly paired to the paired data.

2.2.1.2. The y-homo paired adjusted Ψ^2

The y-homo paired adjusted Ψ^2 , simply adjusted Ψ^2 is the modified y-homo-paired Ψ^2 that has been adjusted to the number of predictors, p in the model. It is used to determine how reliable the correlation is and how much it is determined by the addition of the predictor. In other words, the adjusted Ψ^2 is the statistical tool that measures the goodness-of-fit for HHP multiple regressions by indicating the additional effect of each predictor in the model. The addition of a better predictor increases the value of adjusted Ψ^2 and the addition of a poor predictor in the model reduces the value of adjusted Ψ^2 . Technically, adjusted Ψ^2 offers a small penalty for adding more predictors in the model. If the predictor raises the adjusted Ψ^2 for the homo-hetero pairing regression model, that is a better indication that the predictor has improved the model and can raise the unadjusted Ψ^2 . The adjusted Ψ^2 is expressed as

$$\text{Adjusted } \Psi^2 = 1 - \frac{SSR/(n-p-1)}{SST/(n-1)} = 1 - \frac{n-1}{n-p-1} (1 - \Psi^2)$$

For HHP simple regression, the equation reduced to

$$\text{Adjusted } \Psi^2 = 1 - \frac{n-1}{n-2} (1 - \Psi^2)$$

Where n is the sample size or the number of observations, p is the number of predictors in the model, therefore, $n - p - 1$ is the degree of freedom of the model with sample size, n and number of predictors, p .

2.2.1.3 The y-homo paired root mean square deviation or error

The y-homo paired root mean square deviation or error (MSE) or (RMSE) is another goodness-of-fit statistic that indicates the predictive power of the homo-hetero pairing regression model. It presents the aggregated magnitude of the predictive errors, i.e., the difference between the predicted value and actual values of y-homo pairing changes. In general, the y-homo paired root MSE, or simply root MSE measures the overall accuracy of the predictive model. The lower the y-homo paired root MSE the better as indicates that the predicted values are closest to the actual values, hence the minimal error. The y-homo paired root MSE is the square root of mean square error. That is, the y-homo paired RMSE is the square root of SSE divided by the degree of freedom ($n - p$) of the model; where n is the sample size and p is the parameter to be estimated, for HHP simple regression $p = 2$, which are $\hat{\lambda}_0$ and $\hat{\lambda}_1$. That is,

$$\text{Root } MSE_H = \sqrt{\frac{SSE}{n-p}} = \sqrt{\frac{\sum_{i=1}^n (\partial_{y_i} - \hat{\partial}_{y_i})^2}{n-p}} = \sqrt{\frac{\sum_{i=1}^n (\partial_{y_i} - \hat{\partial}_{y_i})^2}{n-2}}$$

2.2.1.4. The homo-hetero paired confidence interval for lambda coefficients

The homo-hetero paired confidence interval or simply confidence interval provides a means of quantifying the uncertainty produced by the sampling error. The rule of thumb is the same; the 95 percent confidence interval (CI) for the *lambda* coefficient is given by

$$CI = \hat{\lambda}_i \pm 2SE$$

If we assume the predictive model yield unbiased estimates of the parameter $\hat{\lambda}_i$, $\hat{\lambda}_i = \lambda_i$, but if there is a 95 percent chance that our estimate $\hat{\lambda}_i$ will lie with the 2 standard error (SE) of its mean values, λ_i . The standard error of $\hat{\lambda}_i$ or $Se(\hat{\lambda}_i)$ is given,

$$Se(\hat{\lambda}_i) = \sqrt{\frac{SE}{\sum (\partial_{x_i} - \bar{\partial}_x)^2}}$$

The larger is $Se(\hat{\lambda}_i)$, the larger will be the range values, i.e., the wider the confidence interval, and the larger SE, the larger will be $Se(\hat{\lambda}_i)$, and so the wider the CI for the true lambda coefficient. The higher degree of variation of ∂_{x_i} is due to a smaller $Se(\hat{\lambda}_i)$ (tighter confidence interval). The more ∂_{x_i} has varied in the sample, the better the chance we have of accurately picking up any relationship that exists between the *x*-homo pairing changes (∂_{x_i}) and *y*-homo pairing change (∂_y).

2.2.1.5 The homo-hetero paired t-test

The homo-hetero paired t-statistic t_ψ is another measure of the significance of the model. In other words, the homo-hetero paired t-test or simply paired t-test is a method used to test whether the mean difference between pairs of measurements is zero or not. It is applied when the data are separated for “before” and “after” measurements for each pair and need to calculate the differences. The paired t-test is used to test when the data values are paired measurements (Moore, et al. 2013). The assumptions of the use of paired t-tests are subject must be independent, each of the paired measurements must be obtained from the same subject and the measured differences are normally distributed (Moore, et al. 2013). Paired t-test assumes that the changes of observed variables *x* and *y* are linearly pairing or matched variables. Therefore, the appropriate statistical significance test is the *homo-hetero paired t-test*, modified from the traditionally paired t-test (Moore, et al. 2013). That is,

$$t_\psi = \frac{\sum (\partial_{x_i} - \partial_{y_i}) \sqrt{n-1}}{\sqrt{n \sum (\partial_{x_i} - \partial_{y_i})^2 - (\sum (\partial_{x_i} - \partial_{y_i}))^2}} \sim t(n-1)df$$

2.2.1.6. The y-Homo paired F-statistics

The y-homo paired F-test, or simply F-test is the statistical significance test for the homo-hetero pairing regression model, whether any of the predictors are significant (not equal to zero). In other words, the y-homo paired F-tests aimed to provide statistical evidence that the lambda coefficient of the homo-hetero pairing regression model is not zeros. It is given as the ratio of the explained variance and unexplained variance of the y-homo pairing changes, that is,

$$F_H - test = \frac{MSR}{MSE}$$

Where MSR is the mean of squares for regression, which is given by the expression $MSR = \frac{SSR}{p-1}$ and MSE , is the mean of square errors, which is given by the expression $MSE = \frac{SSE}{n-p}$, $p - 1$ is the degree of freedom for regression (model) with p number of predictors, and $n - p$ is the degree of freedom for error with sample size n and the number of predictors p . Therefore,

$$\begin{aligned} F_H - test &= \frac{MSR}{MSE} = \left(\frac{SSR}{p-1} \right) \times \left(\frac{n-p}{SSE} \right) \\ &= \left(\frac{SSR}{SSE} \right) \times \left(\frac{n-p}{p-1} \right) \\ &= \left(\frac{SSR}{SSE} \right) \times \left(\frac{n-p}{p-1} \right) \\ &= \frac{\sum_{i=1}^n (\hat{\partial}_{yi} - \bar{\partial}_y)^2}{\sum_{i=1}^n (\partial_{yi} - \hat{\partial}_{yi})^2} \times \left(\frac{n-p}{p-1} \right) \end{aligned}$$

For simple regression, the formula is reduced to, $p = 2$,

$$F_H - test = \frac{\sum_{i=1}^n (\hat{\partial}_{yi} - \bar{\partial}_y)^2}{\sum_{i=1}^n (\partial_{yi} - \hat{\partial}_{yi})^2} (n - 2)$$

2.3. The HHP Multiple regression modelling

Considering the expanded form of a homo-hetero pairing multiple regression model,

$$\partial_{\bar{y}} = \lambda_0 + \lambda_1 \partial_{x_1} + \lambda_2 \partial_{x_2} + \lambda_3 \partial_{x_3} + \cdots + \lambda_{n-1} \partial_{x_{n-1}} + \lambda_n \partial_{x_n} + \varepsilon$$

This can be written in condensed form, that is,

$$\partial_{\bar{y}} = \lambda_0 + \sum_{i=1}^n \lambda_i \partial_{x_i} + \varepsilon_i$$

$$\varepsilon = \partial_{\bar{y}} - \hat{\partial}_{\bar{y}}$$

Where $\partial_{\bar{y}}$ is a pivotal y-homo pairing change and can be written as,

$$\partial_{\bar{y}_i} = \frac{(y_n - y_{n-1}) - (K_n - K_{n-1})}{(y_n - y_{n-1})}$$

Where K is the pairing means of the independent variables, it is given as,

$$K_i = \frac{1}{p} \sum_{i=1}^p IV_i$$

Where p the number of predictors (independent variable) in the HHP multiple regression model and IV_i is the individual values of predictor (independent variable) in the HHP multiple regression models.

2.4. The HHP regression model and Bundala pairing correlation coefficient

We notice that $\hat{\partial}_{y_i} = \hat{\lambda}_0 + \hat{\lambda}_1 \partial_{x_i}$ is obtained from minimising $\sum_i (\partial_{y_i} - \hat{\lambda}_0 + \hat{\lambda}_1 \partial_{x_i})^2$, and

$\hat{\partial}_{x_i} = \hat{\alpha}_0 + \hat{\beta}_1 \partial_{y_i}$ is obtained from the minimisation of $\sum_i (\partial_{x_i} - \hat{\alpha}_0 + \hat{\beta}_1 \partial_{y_i})^2$, combining the linear equation that $\hat{\partial}_{y_i} = \hat{\lambda}_0 + \hat{\lambda}_1 \partial_{x_i}$ and $\hat{\partial}_{x_i} = \hat{\alpha}_0 + \hat{\beta}_1 \partial_{y_i}$, we get $\hat{\partial}_{x_i} = -\frac{\hat{\lambda}_0}{\hat{\lambda}_1} + \frac{1}{\hat{\lambda}_1} \hat{\partial}_{y_i}$ compares to $\hat{\partial}_{x_i} = \hat{\alpha}_0 + \hat{\beta}_1 \partial_{y_i}$ we get the relations that $\hat{\alpha}_0 = -\frac{\hat{\lambda}_0}{\hat{\lambda}_1}$ and $\hat{\beta}_1 = \frac{1}{\hat{\lambda}_1}$, since there is a relationship between two regression coefficients $\hat{\beta}_1$ and $\hat{\lambda}_1$, and we noticed that,

$$\hat{\lambda}_1 = \frac{\sigma_{\partial_{x_i} \partial_{y_i}}}{\sigma_{\partial_{x_i}}^2},$$

$$\text{Then, } \hat{\beta}_1 = \frac{\sigma_{\partial_{x_i} \partial_{y_i}}}{\sigma_{\partial_{y_i}}^2}$$

Therefore, a product of $\hat{\lambda}_1$ and $\hat{\beta}_1$ should be equal to one, if the two coefficients are perfectly paired and correlated, $\Psi^2 = 1$. That is,

$$\hat{\lambda}_1 \cdot \hat{\beta}_1 = \left(\frac{\sigma_{\partial_{x_i} \partial_{y_i}}}{\sigma_{\partial_{x_i}}^2} \right) \cdot \left(\frac{\sigma_{\partial_{x_i} \partial_{y_i}}}{\sigma_{\partial_{y_i}}^2} \right) = \frac{\sigma_{\partial_{x_i} \partial_{y_i}}^2}{\sigma_{\partial_{x_i}}^2 \cdot \sigma_{\partial_{y_i}}^2}$$

$$\hat{\lambda}_1 \cdot \hat{\beta} = \frac{\sigma_{\partial_{x_i} \partial_{y_i}}^2}{\sigma_{\partial_{x_i}}^2 \cdot \sigma_{\partial_{y_i}}^2} = \Psi^2$$

The Ψ^2 is the same as R^2 in the Pearson correlation.

2.5. The HHP regression model as a data classifier

The HHP regression model can be used as the “classifier” of the structured and unstructured. The classifier is the algorithm that maps or assigns the input data to a specific category. The classification is done to categorise the data into a certain or given number of classes that the new data can fall under. Therefore, the classification model predicts the class label or categories for the new data. The HHP regression model can be applied in the classification modelling of the data. The model can detect and separate the minor homo pairing changes of the paired data x and y which cannot be separated in the normal or traditional linear regression. When the data are closely paired they cannot be separated or categorised into attribute classes. To separate the data we introduce the concept of *homo-hetero pairing scattering*. The homo-hetero pairing changes of x and y are scattered together to identify the classes of pairing behaviour (patterns) of the data. Moreover, the classes can be obtained or established with the application of variable pairing ratios (τ) of the interval of the original data. That is,

$$\tau = \frac{D_min_d_x}{D_min_d_y} \leq \tau \leq \frac{D_max_d_x}{D_max_d_y}$$

Where $D_min_d_x$ and $D_min_d_y$ are sampled data of variables x and y which corresponds to minimum values d_x and d_y respectively. Similarly, $D_max_d_x$ and $D_max_d_y$ are the sampled data of the variables x and y which corresponds to maximum values of d_x and d_y respectively. The τ can be used to classify or assign the new input data in the model by allocating the variable or data to its variable/data pairing ratio interval class. From this concept, each class has a specific variable pairing ratio interval. The ratio can be positive or negative but its sign does not distort the meaning, only the magnitude is considered for the evaluation of the classification model.

2.6. The HHP regression model as a non-linear predictive model

From the classification concept, we notice that the homo-hetero pairing changes can be classified into four homo-hetero classes of paired data. These classes of data are the positive-positive pairing (PPP) class, negative-positive pairing (NPP) class, positive-negative pairing (PNP) class, and negative-negative pairing (NNP) class. Assume each class is bounded or described by the non-linear function relationship between variable x and y by the curve (Figure 2).

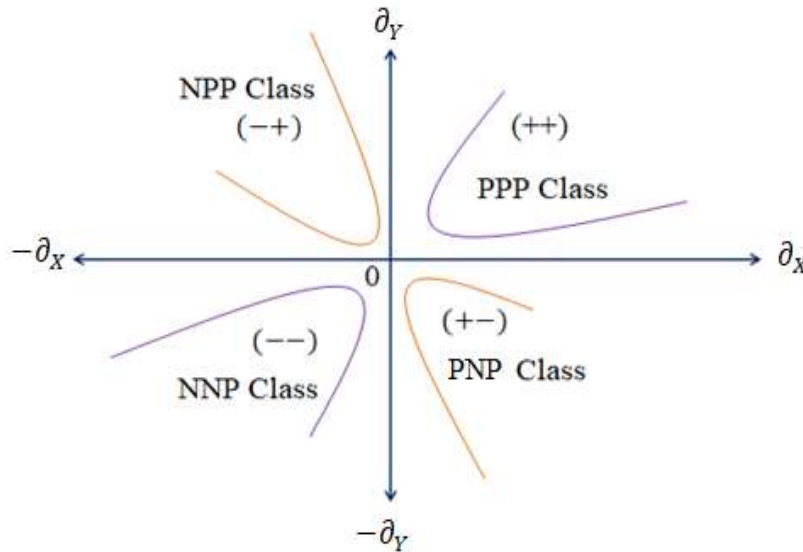


Figure 2: The pairing classes of the homo-hetero non-linear paired data
Source: Author (2022).

Figure 2 shows the four homo-hetero pairing classes of the non-linear paired data. The classes of the non-linear data are termed “quadrant bounded/pairing classes”. Each class is bounded or defined initially by the lowest non-linear function of the paired data – the quadratic curve/function.

$$\partial_y = \lambda_0 + \lambda_1 \partial_x + \lambda_2 \partial_x^2$$

Multiply the lambda coefficient of the equation by the squares of inverse factors of the independent variable (the lowest non-linear inverse factor, $\frac{1}{\partial_x^2}$) to restrict the pairing relationship in the one quadrant (homo-hetero pairing). That is,

$$\partial_{y_p} = \lambda_0 + \frac{\lambda_1 \partial_x}{\partial_x^2} + \frac{\lambda_2 \partial_x^2}{\partial_x^2}$$

$$\partial_{y_p} = \lambda_0 + \lambda_2 + \frac{\lambda_1}{\partial_x}$$

Where, λ_2 is the non-linear accelerating factor or quadrant coefficient or the coefficient (slope) of the second-degree term. This value determines the non-linear pairing attributes of the class. At the perfect pairing classification, $\lambda_2 = 0$, implies that the classes are bounded with quadrant function (inverse functions). That is,

$$\partial_{y_p} = \lambda_0 + \frac{\lambda_1}{\partial_x}$$

Alternatively, the inverse function can be obtained from the HHP simple regression model, when the data are assumed to be perfectly classified by the liner homo-hetero pairing equation, That is, $\lambda_2 = 0$, and multiplied linear HHP regression model by $\frac{1}{\partial_x^2}$

$$\begin{aligned}\partial_Y &= \lambda_0 + \lambda_1 \partial_x \\ \partial_{Y_P} &= \lambda_0 + \frac{\lambda_1 \partial_x}{\partial_x^2} = \lambda_0 + \frac{\lambda_1}{\partial_x}\end{aligned}$$

$$\text{when, } \ln \partial_{Y_P} \approx \partial_{Y_P}$$

The equation becomes an S-curve function/model that is expressed as

$$\ln \partial_{Y_P} = \lambda_0 + \frac{\lambda_1}{\partial_x}$$

$$\partial_{Y_P} = e^{\lambda_0 + \frac{\lambda_1}{\partial_x}}$$

The mathematical modelling of the quadrant pairing classifier was done for four classes of the homo-hetero pairing classes. The classes PPP, NNP, NPP, and PNP are mathematically defined by the homo-hetero inverse function in its four-quadrant bounded (paired) equations (Figure 3).

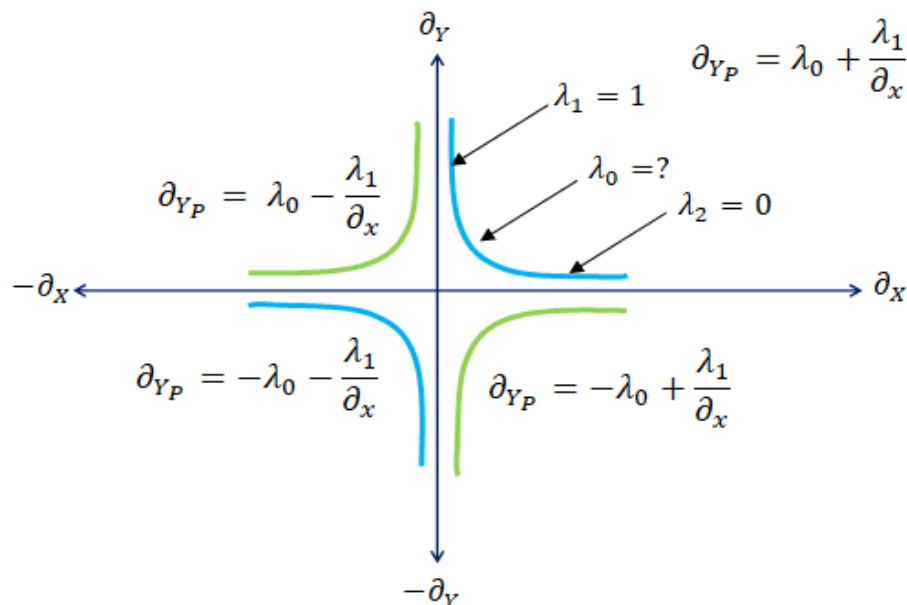


Figure 3: The non-linear equation of the pairing classed of the homo-paired data
Source: Author (2022).

Figure 3 shows the class classifier model of each class. The figure describes the four homo-hetero classes of the paired data. That is,

$$\partial_{Y_P} = \lambda_0 + \frac{\lambda_1}{\partial_x} ; \text{PPP classifier model}$$

$$\partial_{Y_P} = -\lambda_0 - \frac{\lambda_1}{\partial_x} ; \text{NNP classifier model}$$

$$\partial_{Y_P} = \lambda_0 - \frac{\lambda_1}{\partial_x} ; \text{NPP classifier model}$$

$$\partial_{Y_P} = -\lambda_0 + \frac{\lambda_1}{\partial_x} ; \text{PNP classifier model}$$

If the equation is not restricted to the quadrant pairing classes function, the equation can take higher functions or degrees such as quadratic, cubic, etc. That is,

$$\partial_Y = \lambda_0 + \lambda_1 \partial_x + \lambda_2 \partial_x^2 + \dots + \lambda_{n-1} \partial_x^{2n-1} + \lambda_n \partial_x^n$$

If the paired data take a higher degree function (polynomial regression), they describe the multiple informational changes or influences. In other words, the prediction of a stock price is highly influenced by multiple external and internal factors. The HHP polynomial regression has no clear or definitive classes; the data are either paired in more than one or two quadrants (Figure 4).

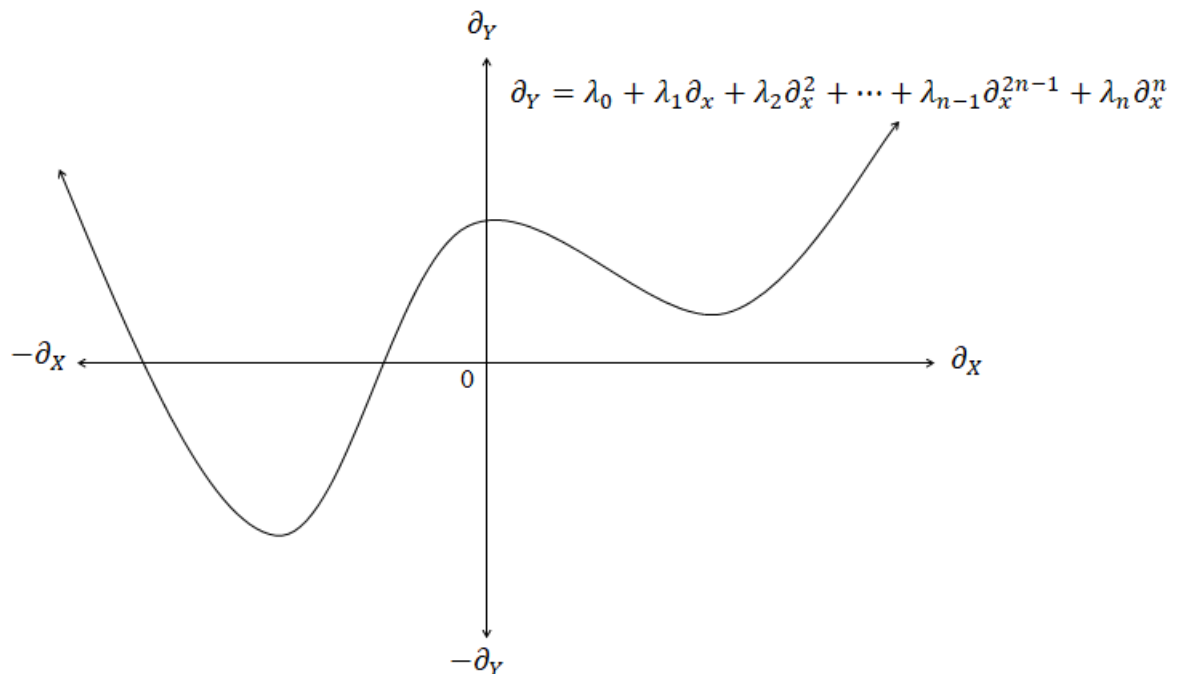


Figure 4: The HHP polynomial regression model

Source: Author (2022).

Figure 4 describes the general non-linear equation of the quadrant unrestricted classes function". This implies that the data can be defined in the quadratic function, cubic function, etc. The HHP polynomial regression is the best predictive model because it captures the general trending behaviour of the paired data (Gururay et al., 2019; Demiray and Gurhanli, 2021; Karim et al., 2021; Sahoo and Charlapally, 2015).

3. Research Method

The study aimed to examine the technical and fundamental hypotheses in the New York Stock Exchange Market (NYSE), Standard and Poor's 500 (S&P 500) and the National Association of Securities Dealers Automatic Quotation System (NASDAQ) stock exchange markets. The Homo-Hetero Pairing Regression Model, simply the HHP regression model was used. This model was introduced to detect the linear and non-linear data, hence being an appropriate econometric predictive model for homo paired data. The data were collected from the three largest stock market exchanges in the world which are the New York Stock Exchange Market (NYSE), Standard and Poor's 500 (S&P 500) and the National Association of Securities Dealers Automatic Quotation System (NASDAQ). The 240 days' daily closing prices and trading volumes data at NYSE were systematically collected from 4, June 2021 to 3, June 2022. Moreover, the closing price and trading volume data for 197 days were collected from 4, June 2021 to 2, June 2022, and the 253 days' closing price and trading volume data of NASDAQ were systematically collected from 4 June 2021 to 3 June 2022.

The independent variable of the first objective (fundamental analysis) is defined as the daily information available at the stock exchange market and measured by homo-pairing changes in days of trading. The dependent variable is defined as the daily closing stock price level and measured by homo-pairing changes of the daily stock closing price level. The variables of the study are measured as follows,

The change of daily information is the independent variable proxies as homo-pairing changes in days of trading, d with a given closing price P is given by,

$$\partial_{DP} = \frac{(d_2 - d_1) - (P_2 - P_1)}{d_2 - d_1}$$

The dependent variable is the change of the daily closing stock price measured in the homo pairing changes in daily closing stock price, P with days of trading, d , it is given as,

$$\partial_{PD} = \frac{(P_2 - P_1) - (d_2 - d_1)}{P_2 - P_1}$$

The second, objective independent variable is measured by the homo-pairing changes in daily stock closing prices P , with a given daily trading volume V ; it is expressed as,

$$\partial_{PV} = \frac{(P_2 - P_1) - (V_2 - V_1)}{P_2 - P_1}$$

The dependent variable is the changes of the daily trading volumes measured by homo-pairing changes in daily trading volumes V , with a given daily closing stock price, P . It is expressed as,

$$\partial_{VP} = \frac{(V_2 - V_1) - (P_2 - P_1)}{V_2 - V_1}$$

The tests of linear assumptions were done. The Breusch-Pagan/Cook-Weisberg test for heteroscedasticity was conducted and found to have an F-value of 0.63 with a p-value of 0.6564 which is greater than the recommended critical value of the significance level of 0.05; this indicates that the data do not have a constant variance; the null hypothesis was rejected at 95 percent of confidence level. On the hand, the normality test was done by using the Jarque-Bera normality test and found to be 5.381 and Chi (2) is 0.0678) which indicates that the data are normally distributed (Thadewald and Herbert, 2004). Furthermore, the multicollinearity test was done by using variance inflation factors (VIF) and found to have a mean value of VIF of 1.38 which is good as it is below 5 (Murray, Nguyen, Lee...and David, 2012). Moreover, the reliability test was done by using Cronbach alpha and found to be 0.6775 which is reasonably better as suggested by Tavakol and Dennick (2011).

4. Results and discussion

4.1 Descriptive statistics

The study specifically aimed to find out the operational behaviour of the three stock exchange markets, S&P 500, NYSE and NASDAQ which are among the largest stock exchange market in the world. The data was analysed by using the newly introduced HHP regression model that can detect both linear and non-linear pairing effects and impacts of the paired data such as stock prices and trading volumes. The descriptive statistics are displayed to show the general characteristics of the three markets (Table 2).

Table 2: The Descriptive statistics for daily closing stock prices and trading volumes

Variable	Obs	Mean	Std. Dev.	Min	Max
Price_S&P 500	197	4421.932	176.8439	3900.79	4796.56
Volume_S&P 500	197	3.41e+09	5.34e+08	2.37e+09	4.75e+09
Price_NASDAQ	219	14331.83	1136.294	11264.45	16057.44
Volume_NASDAQ	219	4.74e+09	7.31e+08	3.38e+09	8.11e+09
Price_NYSE	240	16543.59	498.9996	14902.14	17353.76
Volume_NYSE	240	3.50e+09	7.34e+08	2.19e+09	6.68e+09
$\partial_{PV_S\&P500}$	196	2023092	3.30e+07	-1.46e+08	1.25e+08
$\partial_{VP_S\&P500}$	196	1	3.13e-07	.9999995	1.000001
∂_{PV_NASDAQ}	218	1343238	6080752	-1.46e+07	2.54e+07
∂_{VP_NASDAQ}	218	1	2.62e-06	.99999	1.000013
∂_{PV_NYSE}	239	812524.2	1.31e+07	-6.07e+07	6.28e+07
∂_{VP_NYSE}	239	.9999999	1.88e-06	.9999879	1.000005

Source: Author (2022).

Table 2 shows the descriptive statistics of daily stock prices and trading volumes of the S&P 500, NYSE and NASDAQ stock markets. The average daily stock closing price of S&P 500 stock for 197 days is 4421.932 US\$ million and varies from 3900.79 to 4796.56 US\$ Million. The daily stock trading volume is averaged at 3.41e+09 US\$ million and varies from 2.37e+09 to 4.75e+09 US\$ million. On the other hand, the daily closing stock price of the NASDAQ stock for 219 days is 14331.83US\$Million and has a range of a minimum of 11264.45 to a maximum of 16057.44 US\$ million, its corresponding daily stock trading volume is averaged at 4.74e+09 US\$ million and ranges from 3.38e+09 to 8.11e+09 US\$ million. Furthermore, the daily closing stock price of the NYSE for 240 days is averaged at 16543.59 US\$ million and has a range of minimum value of 14902.14 to a maximum value of 17353.76 US\$ million, and its corresponding daily stock trading volume is average at 3.503e+09 and ranges from minimum 2.19e+09 to maximum value of 6.68e+09 US\$ million.

On the other hand, the mean homo-pairing changes in daily closing stock price relative to its trading volume in S&P 500 stock exchange market is 2023092, which ranges from -1.46e+08 to 1.25e+08. The mean homo-pairing changes in daily trading volume relative to its daily closing stock price in the S&P500 stock exchange market is 1 which ranges from 0.9999995 to 1.000001. The mean homo-pairing changes in the daily closing stock price relative to daily trading volume in NASDAQ stock exchange markets is 1343238 which ranges from -1.46e+07 to 2.54e+07, and its mean homo-pairing changes in daily trading volume relative to daily closing stock price are 1 and range from 0.99999 to 1.000013. Moreover, the mean homo-pairing changes in the daily closing stock price relative to daily trading volume in NYSE is 812524.2 and has a range from -6.07e+07 to 6.28e+07, and its mean homo pairing changes in daily trading volume relative to the daily closing stock price is 0.9999999 and ranges from 0.9999879 to 1.000005. From the descriptive statistics, we learn that there is an extremely “daily variation” of the mean homo-pairing changes in the daily closing stock price relative to daily trading volume. On the other hand, there is a slight variation in the mean homo-pairing changes in daily trading volume relative to the daily closing stock price.

The descriptive statistics for mean homo-pairing changes in daily stock information relative to daily closing stock price and mean homo-pairing changes in daily closing stock price relative to daily stock information were provided (Table 3).

Table 3: The Descriptive statistics for daily stock information and closing stock prices

Variable	Obs	Mean	Std. Dev.	Min	Max
∂_{PD_SP}	196	1.013207	.2070717	-.1111111	2.754386
∂_{DP_SP}	196	.7292347	57.49828	-307.93	229.54
∂_{PD_NASD}	252	1.017135	.562253	-3.347826	8.692308
∂_{DP_NASD}	252	-6.149841	213.8699	-646.17	488.93
∂_{PD_NYSE}	239	.9942284	.0499585	.5260664	1.102459
∂_{DP_NYSE}	239	-2.8141	167.191	-507.69	432.97

Source: Author (2022).

Table 3 shows the descriptive statistics for fundamental hypothesis analysis in S&P 500, NYSE and NASDAQ stock exchange markets. The mean homo-pairing changes in daily stock information relative to daily closing stock price and the mean homo-pairing changes in daily closing stock price relative to the daily stock information. The mean homo-pairing changes in daily closing stock price relative to daily stock information in the S&P500 stock exchange market is 1.013207 with a range value from -0.1111111 to 2.754386. The mean homo-pairing changes in daily stock information relative to the daily closing stock price are 0.7292347 and range from -307.93 to 229.54. The mean homo-pairing changes in daily closing stock price relative to daily stock information in the NASDAQ stock exchange market is 1.0117135 and has the range from -3.347826 to 8.692308, and its mean homo-pairing changes in daily stock information relative to daily closing stock price are -6.149841 and have ranged from -646.17 to 488.93. And, the mean homo- pairing changes in daily closing stock price relative to daily stock information in the NYSE is 0.9942284 and ranges from 0.5260664 to 1.102459, and has to mean homo- pairing changes in daily stock information relative to the daily closing stock price of - 2.8141 and ranges from -507.69 to 432.97.

4.2 Technical analysis in the S&P 500 Stock market

The technical analysis is objectively done by examining the statistical records available, particularly the daily closing stock price and the daily trading volumes in S&P 500 stock exchange market. In other words, the study examined how the statistical information or data can convey or signals the stock price level in the S&P 500 stock exchange market. The linear and non-linear pairing impact of the homo pairing change in daily trading volume relative to a change in daily closing stock price, ∂_{VP} on the homo pairing changes in daily closing stock price relative to change of daily trading volume, ∂_{PV} was examined by using the HHP regression model. The study hypothesis (alternative hypothesis) is that the homo pairing change in daily trading volume relative to the change in daily closing stock price has a non-linear pairing impact on the homo pairing change in daily closing stock price relative to the change in daily trading volume in the S&P500 stock exchange market. The HHP regression model was run using IBM SPSS software and the model summary and parameter estimates were provided (Table 4).

Table 4: Model summary and parameter estimates in the S&P 500 stock exchange market

Dependent Variable: Relative price-homo pairing changes in daily stock trading volume (∂_{VP})							
Model Summary					Parameter Estimates		
Equation	Ψ^2	F	df1	df2	Sig.	Constant (λ_0)	λ_1
Linear	0.007	1.287	1	194	0.258	1.000	7.690E-16
Inverse	1.000	692312462527441.6	1	194	0.000	1.000	1.000
S	1.000	2764868125663971	1	194	0.000	3.016E-14	1.000

The independent variable is Relative volume- homo pairing changes in daily closing stock price (∂_{PV})

Source: Author (2022).

Table 4 shows the HHP regression model summary and its parameter estimates. The table provides the three parameters of the three models; the linear, inverse and S-curved models. The

linear model has a positive lambda coefficient of 7.690E-16 and a constant value of 1.000, a p-value of 0.258 which is greater than the recommended critical value of a significant level of 0.05. The model is poorly determined as it has a linear pairing determination coefficient, Ψ^2 of 0.007. Therefore, we reject the null hypothesis and accept the alternative hypothesis at a 95 percent of confidence level since its p-value of 0.258 is greater than the critical value of 0.05. The second model is the inverse model which has a positive lambda coefficient of 1.000, a non-linear pairing determination coefficient of 1.000 (perfect correlation) and a constant value of 1.000, with a p-value of 0.000 which is less than a recommended critical value of the significant level of 0.05. Hence, we reject the null hypothesis and accept the alternative hypothesis at a 95 percent of confidence level since its p-value of 0.000 is less than the recommended critical value of 0.05. The third model is the S-curved model which has a positive lambda coefficient of 1.000, a constant value of 3.016E-14, a non-linear pairing determination coefficient of 1.000 (perfect correlation) and the p-values of 0.000, which is less than the recommended critical value of the significant level of 0.05. Hence, we reject the null hypothesis and accept the alternative hypothesis at a 95 percent of confidence level since its p-value of 0.000 is less than the recommended critical value of 0.05.

The two significant models-inverse and S-curve models are described in non-linear pairing classes; the PPP class and NPP class. The classes are explained by the homo-hetero scattering plots established by curve estimation (Figure 5). The curve estimation plots the scattering of the homo-pairing changes of the daily closing stock price and the daily stock trading volume to reveal the classes that are described by the non-linear pairing function; the inverse and S-curve models.

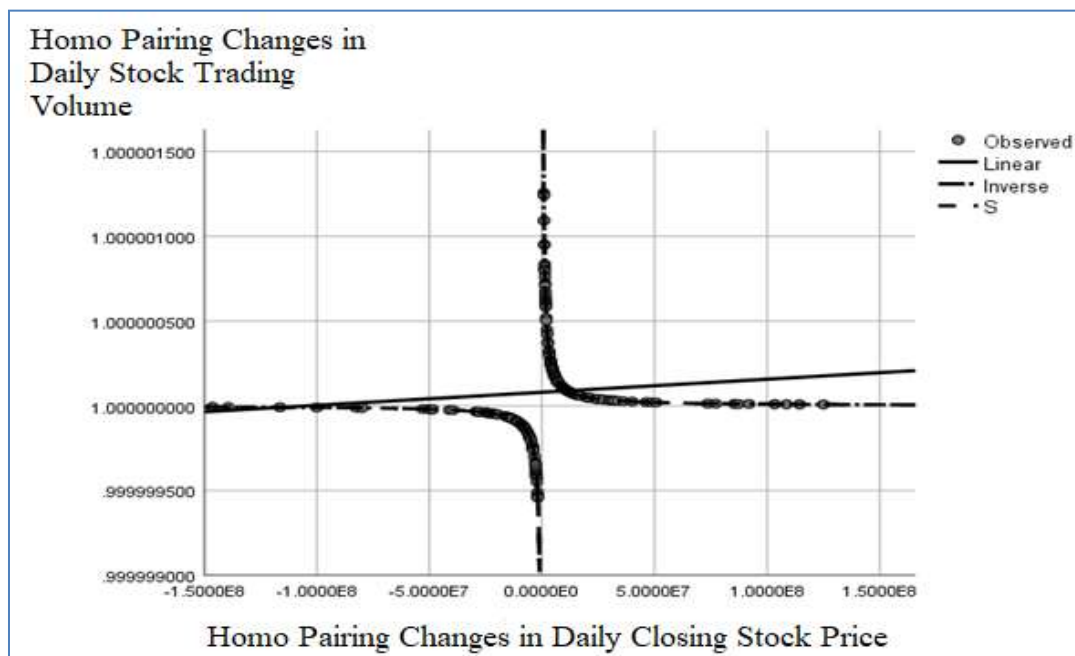


Figure 5: Curve estimation of homo pairing changes of stock price and trading volume
Source: Author (2022).

Figure 5 shows the curve estimation of the homo-pairing changes in the daily trading volume relative to the change in the daily closing stock price in the S&P 500 stock exchange market. The figure exhibits two classes that are described by the non-linear pairing functions; the inverse and S-curved models. Each model is described by two non-pairing classes, the PPP and NPP classes. Therefore, since the inverse model has a lambda coefficient is one, and the constant value is one, the S&P 500 stock exchange market technical model is described as;

$$\partial_{VP} = 1 + \frac{1}{\partial_{PV}} ; \text{ PPP class model}$$

$$\partial_{VP} = 1 - \frac{1}{\partial_{PV}} ; \text{ NPP class model}$$

On the other hand, the S&P 500 stock exchange market technical model can be determined by the S-curved function. Since the S-curved model has a unit lambda coefficient and the constant value of 3.016E-14, therefore, S&P500 stock exchange market technical model is described as;

$$\ln \partial_{VP} = 3.016 \times 10^{-14} + \frac{1}{\partial_{PV}} ; \text{ PPP class model}$$

$$\ln \partial_{VP} = 3.016 \times 10^{-14} - \frac{1}{\partial_{PV}} ; \text{ NPP class model}$$

In conclusion, both empirical models-inverse and S-curved models that represent the S&P500 stock exchange market technical models proved that the homo pairing changes in daily closing stock price relative to the daily trading volume are non-linearly related to the homo pairing changes in daily trading volume relative to the change in the daily closing stock price.

4.3 Fundamental analysis in the S&P 500 Stock exchange market

The fundamental analysis of the S&P 500 stock exchange market was done to examine the impact of the daily stock information available in the market on the stock price. The study assumed that if the change in daily closing price happened due to the change of a day (that next day), therefore the change was done due to the information attained about the specific stock immediately available or collected at the market. That is, the change in trading days relative to the change in closing stock price indicates the “impactful information signal”. The study regressed the homo-pairing changes in daily closing stock price relative to changes in daily stock information, ∂_{PD} to homo pairing change in daily stock information relative to daily closing stock price, ∂_{DP} to determine the linear and non-linear pairing impact of the two paired variables. The study hypothesis (alternative hypothesis) is that the homo pairing change in the daily closing stock price relative to the change in daily stock information available in the stock exchange market has a non-linear pairing impact on the homo pairing changes in the daily stock information available relative to a change in the daily closing stock price in the S&P 500 stock exchange market. The HHP regression model was run using IBM SPSS software and the model summary and parameter estimates were provided (Table 5).

Table 5: Model Summary and Parameter Estimates in S&P 500 stock exchange market

Dependent Variable: Homo Pairing Changes of Closing Stock Price (∂_{PP})							
Equation	Model Summary					Parameter Estimates	
	Ψ^2	F	df1	df2	Sig.	Constant (λ_0)	λ_1
Linear	0.050	9.780	1	186	0.002	1.002	0.000
Inverse	0.969	5776.651	1	186	0.000	1.008	0.909
S	0.992	23142.589	1	186	0.000	0.004	0.947

The independent variable is Homo Pairing Changes of Days of Stock Trading (∂_{DP})

Source: Author (2022).

Table 5 shows the HHP regression summary and parameter estimates for the S&P 500 stock exchange market fundamental model. The table described three fundamental models in the S&P 500 stock exchange market-the linear, inverse and S-curved models. The linear model has a positive lambda coefficient of 0.000, a constant value of 1.002, a linear pairing determination coefficient of 0.050, with a p-value of 0.002, which is less than the recommended critical value of the significant level of 0.05, therefore, we reject the null hypothesis and accept the study hypothesis at a 95 percent of confidence level; although, the linear model is poorly determined at 0.050. The second model is the inverse model that has a positive lambda coefficient of 0.909 and constant value of 1.008, with a non-linear pairing determination coefficient, Ψ^2 of 0.969, and a p-value of 0.000, which is less than the recommended critical value of a significant level of 0.05. Hence, the study rejects the null hypothesis and accepts the alternative hypothesis at a 95 percent of confidence level. The third model is the S-curved model which has a positive lambda coefficient of 0.947, a constant value of 0.004, and a non-linear pairing determination coefficient, Ψ^2 of 0.992, with a p-value of 0.000 which is less than the recommended critical value of the significant level of 0.05, hence, the study rejects the null hypothesis and accepts the alternative hypothesis at a 95 percent of confidence level.

The two significant models- inverse and S-curve models are described in non-linear pairing classes; the PPP class and NPP class. The classes are explained by the homo-hetero scattering plots established by curve estimation (Figure 6). The curve estimation plots the scattering of homo pairing changes in the daily closing stock price relative to the change in daily stock information and the homo pairing changes in daily stock information relative to the change in the daily closing stock price in the S&P500 stock exchange market. The classes are described by the non-linear pairing function-the inverse and S-curve models.

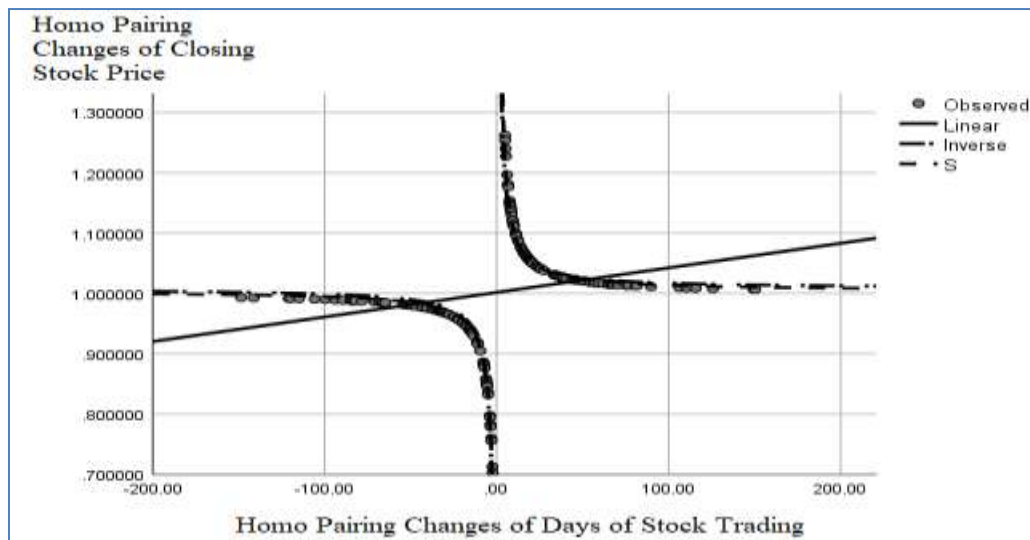


Figure 6: Curve estimation of homo pairing changes in stock price and days of stock trading
Source: Author (2022).

Figure 6 shows the curve estimation of the homo-pairing changes in the daily closing stock price relative to the change in daily stock information and homo pairing change in daily stock information relative to a change in the daily closing stock price in the S&P 500 stock exchange market. The figure exhibits two classes that are described by the non-linear pairing functions; the inverse and S-curved models. Each model is described by two non-pairing classes, the PPP and NPP classes. Therefore, since the inverse model has a lambda coefficient is 0.909, and the constant value is 1.008, the S&P500 stock exchange market fundamental model is described as;

$$\partial_{PD} = 1.008 + \frac{0.909}{\partial_{DP}} ; \text{ PPP class model}$$

$$\partial_{PD} = 1.008 - \frac{0.909}{\partial_{DP}} ; \text{ NPP class model}$$

On the other hand, the S&P stock exchange market technical model can be determined by the S-curved function. Since the S-curved model has a lambda coefficient of 0.947 and the constant value is 0.004, therefore, S&P500 stock exchange market fundamental model is described as;

$$\ln \partial_{PD} = 0.004 + \frac{0.947}{\partial_{DP}} ; \text{ PPP class model}$$

$$\ln \partial_{PD} = 0.004 - \frac{0.947}{\partial_{DP}} ; \text{ NPP class model}$$

In conclusion, both empirical models-inverse and S-curved models that represent the S&P500 stock exchange market fundamental models proved that the homo pairing changes in daily closing stock price relative to changes in daily stock information are non-linearly related to the changes in daily stock information relative to the changes in daily closing stock price.

4.4 Technical analysis in the NASDAQ Stock exchange market

The technical analysis in the NASDAQ stock exchange market was done by regressing the homo-pairing change in the daily closing stock price relative to daily trading volume, ∂_{PV} and the homo-pairing changes in daily trading volume relative to daily closing stock price, ∂_{VP} . The study aimed to examine the linear and non-linear pairing impacting behaviours between the stock prices and trading volume in the NASDAQ stock exchange market. The study hypothesis (alternative hypothesis) is that the homo pairing changes in daily closing stock price relative to the daily stock information have a non-linear pairing impact on the homo pairing change in daily trading volume relative to the change in the daily closing stock price in the NASDAQ stock exchange market. The HHP regression model was run using IBM SPSS software and the model summary and parameter estimates were provided (Table 6).

Table 6: Model Summary and Parameter Estimates in the NASDAQ Stock Exchange Market

Dependent Variable: The relative Price- homo pairing changes in daily stock trading volume, ∂_{VP}							
Model Summary						Parameter Estimates	
Equation	Ψ^2	F	df1	df2	Sig.	Constant (λ_0)	λ_1
Linear	0.002	0.532	1	216	0.467	1.000	2.134E-14
Inverse	1.000	3258307635319.490	1	216	0.000	1.000	1.000
S	1.000	13032401156115.164	1	216	0.000	3.014E-12	1.000

The independent variable is relative volume - homo pairing changes in daily closing stock price, ∂_{PV}

Source: Author (2022).

Table 6 shows the HHP regression model summary and its parameter estimates. The table provides the three parameters of the three models; the linear, inverse and S-curved models. The linear model has a positive lambda coefficient of 2.134E-14 and a constant value of 1.000, a p-value of 0.467 which is greater than the recommended critical value of a significant level of 0.05. The model is poorly determined as it has a linear pairing determination coefficient, Ψ^2 of 0.002. Therefore, we reject the null hypothesis and accept the alternative hypothesis at a 95 percent of confidence level since its p-value of 0.467 is greater than the critical value of 0.05. The second model is the inverse model which has a positive lambda coefficient of 1.000, a non-linear pairing determination coefficient of 1.000 (perfect correlation) and a constant value of 1.000, with a p-value of 0.000 which is less than a recommended critical value of the significant level of 0.05. Hence, we reject the null hypothesis and accept the alternative hypothesis at a 95 percent of confidence level since its p-value of 0.000 is less than the recommended critical value of 0.05. The third model is the S-curved model which has a positive lambda coefficient of 1.00, a constant value of 3.014E-12, a non-linear pairing determination coefficient of 1.000(perfect correlation) and the p-values of 0.000, which is less than the recommended critical value of the significant level of 0.05. Hence, we reject the null hypothesis and accept the alternative hypothesis at a 95 percent of confidence level.

The two significant models-inverse and S-curve models are described in non-linear pairing classes; the PPP class and NPP class. The classes are explained by the homo-hetero scattering plots established by curve estimation (Figure 7). The curve estimation plots the scattering of the

relative price homo-pairing changes in the daily trading volume relative to the change of daily closing stock price and the homo pairing changes in daily closing stock price relative to daily trading volume in to reveal the classes that are described by the non-linear pairing function; the inverse and S-curve models.

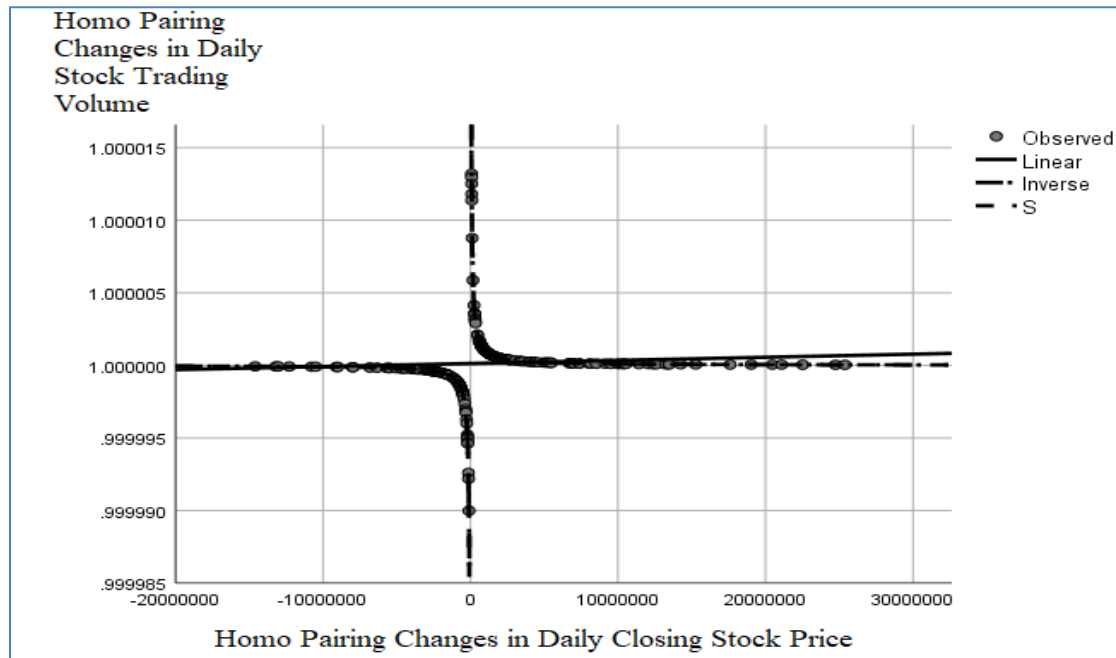


Figure 7: Curve estimation of homo pairing changes in stock price and stock trading volume
Source: Author (2022).

Figure 7 shows the curve estimation of the homo pairing changes in the daily trading volume relative to the change in daily closing stock price and the homo pairing changes in daily closing stock price relative to the change in trading volume in the NASDAQ stock exchange market. The figure exhibits two classes that are described by the non-linear pairing functions; the inverse and S-curved models. Each model is described by two non-linear pairing classes, the PPP and NPP classes. Therefore, since the inverse model has a lambda coefficient is one, and the constant value is one, the NASDAQ stock exchange market technical model is described as;

$$\partial_{VP} = 1 + \frac{1}{\partial_{PV}} ; \text{ PPP class model}$$

$$\partial_{VP} = 1 - \frac{1}{\partial_{PV}} ; \text{ NPP class model}$$

On the other hand, the NASDAQ stock exchange market technical model can be determined by the S-curved function. Since the S-curved model has a unit lambda coefficient and the constant value is 3.014E-12, NASDAQ stock exchange market technical model is described as;

$$\ln \partial_{VP} = 3.014 \times 10^{-12} + \frac{1}{\partial_{PV}} ; \text{PPP class model}$$

$$\ln \partial_{VP} = 3.014 \times 10^{-12} - \frac{1}{\partial_{PV}} ; \text{NPP class model}$$

In conclusion, both empirical models- inverse and S-curved models that represent the NASDAQ stock exchange market technical models, proved that the homo pairing changes in daily trading volume relative to change in daily closing stock price are non-linear related to the homo pairing changes in daily closing stock price relative to the change in daily trading volume in the NASDAQ stock exchange market.

4.5 Fundamental analysis in the NASDAQ Stock exchange market

The fundamental analysis in the NASDAQ stock exchange market was done by regressing the homo-pairing change of the daily closing stock price relative to the change in daily stock information, ∂_{PD} , and the homo pairing changes in daily stock information relative to the change in the daily closing stock price, ∂_{DP} . The study hypothesis (alternative hypothesis) is that the homo pairing changes in daily closing stock price relative to the change in daily stock information have a non-linear pairing impact on the homo pairing changes in daily stock information relative to the change in the daily closing stock price in the NASDAQ stock exchange market. The HHP regression model was run using SPSS software and the model summary and parameter estimates were provided (Table 7).

Table 7: Model Summary and Parameter Estimates in the NASDAQ Stock Exchange Market

Dependent Variable: Relative-days homo pairing changes of daily closing stock prices, ∂_{PD} ,							
Model Summary						Parameter Estimates	
Equation	Ψ^2	F	df1	df2	Sig.	Constant (λ_0)	λ_1
Linear	0.030	6.576	1	216	0.011	1.001	1.896E-5
Inverse	0.990	20628.300	1	216	0.000	1.001	0.986
S	0.997	82132.046	1	216	0.000	0.000	0.991

The independent variable is relative-price homo pairing changes of daily stock information, ∂_{DP} ,

Source: Author (2022).

Table 7 shows the HHP regression summary and parameter estimates for the NASDAQ stock exchange market fundamental model. The table described three fundamental models in the NASDAQ stock exchange market- the linear, inverse and S-curved models. The linear model has a positive lambda coefficient of 1.896E-5, a constant value of 1.001, and a linear pairing coefficient of 0.030, with a p-value of 0.011, which is less than the recommended critical value of the significant level of 0.05; therefore, we reject the null hypothesis and accept the study hypothesis at a 95 percent of confidence level; although, the linear model is poorly determined at 0.030. The second model is the inverse model that has a positive lambda coefficient of 0.986 and constant value of 1.001, with a non-linear pairing determination coefficient, Ψ^2 of 0.990, and a p-value of 0.000, which is less than the recommended critical value of a significant level of 0.05. Hence, the stud rejects the null hypothesis and accepts the alternative hypothesis at a 95 percent

of confidence level. The third model is the S-curved model which has a positive lambda coefficient of 0.991, a constant value of 0.000(3dp), and a non-linear pairing coefficient, Ψ^2 of 0.997, with a p-value of 0.000 which is less than the recommended critical value of the significant level of 0.05, hence, the study rejects the null hypothesis and accepts the alternative at a 95 percent of confidence level.

The two significant models- inverse and S-curve models are described in non-linear pairing classes; the PPP class and NPP class. The classes are explained by the homo-hetero scattering plots established by curve estimation (Figure 8). The curve estimation plots the scattering of homo pairing changes in the daily closing stock price relative to the changes in daily stock information and the homo pairing changes in daily stock information relative to the change in the daily closing stock price in the NASDAQ stock exchange market. The classes are described by the non-linear pairing function-the inverse and S-curve models.

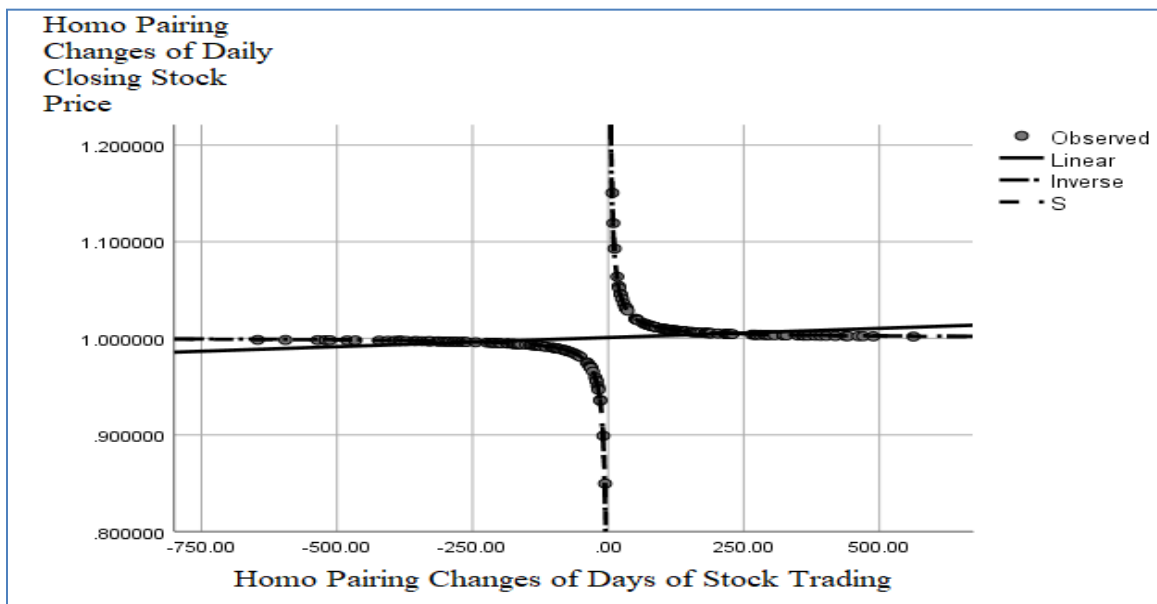


Figure 8: Curve estimation of homo pairing changes in stock price and days of stock trading

Figure 8 shows the curve estimation of the homo pairing changes of the daily closing stock price relative to the change of daily stock information and the homo pairing changes in daily stock information relative to the change in the daily closing stock price in the NASDAQ stock exchange market. The figure exhibits two classes that are described by the non-linear pairing functions- the inverse and S-curved models. Each model is described by two non-pairing classes, the PPP and NPP classes. Therefore, since the inverse model has a lambda coefficient is 0.986, and the constant value is 1.001, the NASDAQ stock exchange market fundamental model is described as;

$$\partial_{PD} = 1.001 + \frac{0.986}{\partial_{DP}} ; \text{ PPP class model}$$

$$\partial_{PD} = 1.001 - \frac{0.986}{\partial_{DP}} ; \text{NPP class model}$$

On the other hand, the NASDAQ stock exchange market fundamental model can be determined by the S-curved function. Since the S-curved model has a λ coefficient of 0.991 and the constant value is 0.000(dp), the NASDAQ stock exchange market fundamental model is described as;

$$\ln \partial_{PD} = 0.000 + \frac{0.991}{\partial_{DP}} ; \text{PPP class model}$$

$$\ln \partial_{PD} = 0.000 - \frac{0.991}{\partial_{DP}} ; \text{NPP class model}$$

In conclusion, both empirical models- inverse and S-curved models that represent the NASDAQ stock exchange market fundamental models proved that the homo pairing changes in the daily closing stock price relative to the daily stock information are non-linearly related to the homo pairing changes in daily stock information relative to the daily closing stock price.

4.6 Technical analysis in the NYSE market

The technical analysis in the NYSE market was done by regressing the homo-pairing changes in daily trading volume relative to the change in the daily closing stock price, ∂_{VP} , and the homo pairing changes daily closing stock price relative to the change in the daily trading volume, ∂_{PV} . The study hypothesis (alternative hypothesis) is that the homo pairing changes in daily trading volume relative to the change in daily closing stock price have a non-linear pairing impact on the homo pairing change in daily closing stock price relative to the daily trading volume in the NYSE market. The HHP regression model was run using IBM SPSS software and the model summary and parameter estimates were provided (Table 8).

Table 8: Model Summary and Parameter Estimates NYSE market

Dependent Variable: Relative –price homo pairing changes in daily stock trading volume (∂_{VP})							
Equation	Model Summary					Parameter Estimates	
	Ψ^2	F	df1	df2	Sig.	Constant (λ_0)	λ_1
Linear	0.002	0.467	1	237	0.495	1.000	6.370E-15
Inverse	1.000	8106027854060.792	1	237	0.000	1.000	1.000
S	1.000	32425588956436.074	1	237	0.000	1.451E-12	1.000

The independent variable is relative-volume homo pairing changes in daily closing stock price (∂_{PV})

Source: Author (2022).

Table 8 shows the HHP regression model summary and its parameter estimates. The table provides the three parameters of the three models; the linear, inverse and S-curved models. The linear model has a positive λ coefficient of 6.370E-15 and a constant value of 1.000, a p-value of 0.495 which is greater than the recommended critical value of a significant level of 0.05. The model is poorly determined as it has a linear pairing determination coefficient, Ψ^2 of 0.002. Therefore, we reject the null hypothesis and accept the alternative hypothesis at a 95 percent of confidence level since its p-value of 0.467 is greater than the critical value of 0.05. The second

model is the inverse model which has a positive lambda coefficient of 1.000, a non-linear pairing coefficient, Ψ^2 of 1.000 (perfect correlation) and a constant value of 1.000, with a p-value of 0.000 which is less than the recommended critical value of the significant level of 0.05. Hence, we reject the null hypothesis and accept the alternative hypothesis at a 95 percent of confidence level since its p-value of 0.000 is less than the recommended critical value of 0.05. The third model is the S-curved model which has a positive lambda coefficient of 1.00, a constant value of 1.451E-12, a non-linear pairing coefficient, Ψ^2 of 1.000(perfect correlation) and the p-values of 0.000, which is less than the recommended critical value of the significant level of 0.05. Hence, we reject the null hypothesis and accept the alternative hypothesis at a 95 percent of confidence level.

The two significant models- inverse and S-curve models are described in non-linear pairing classes; the PPP class and NPP class. The classes are explained by the homo-hetero scattering plots established by curve estimation (Figure 9). The curve estimation plots the scattering of homo pairing changes in the daily trading volume relative to the change in daily the closing stock price and the homo pairing changes in the daily closing stock price relative to the change in the daily trading volume in the NYSE market. The classes are described by the non-linear pairing function-the inverse and S-curve models.

Figure 9 shows the curve estimation of the homo pairing changes of the daily trading volume relative to the daily closing stock price and the homo pairing changes in the daily stock trading volume relative to the change in the daily trading volume in the NYSE market

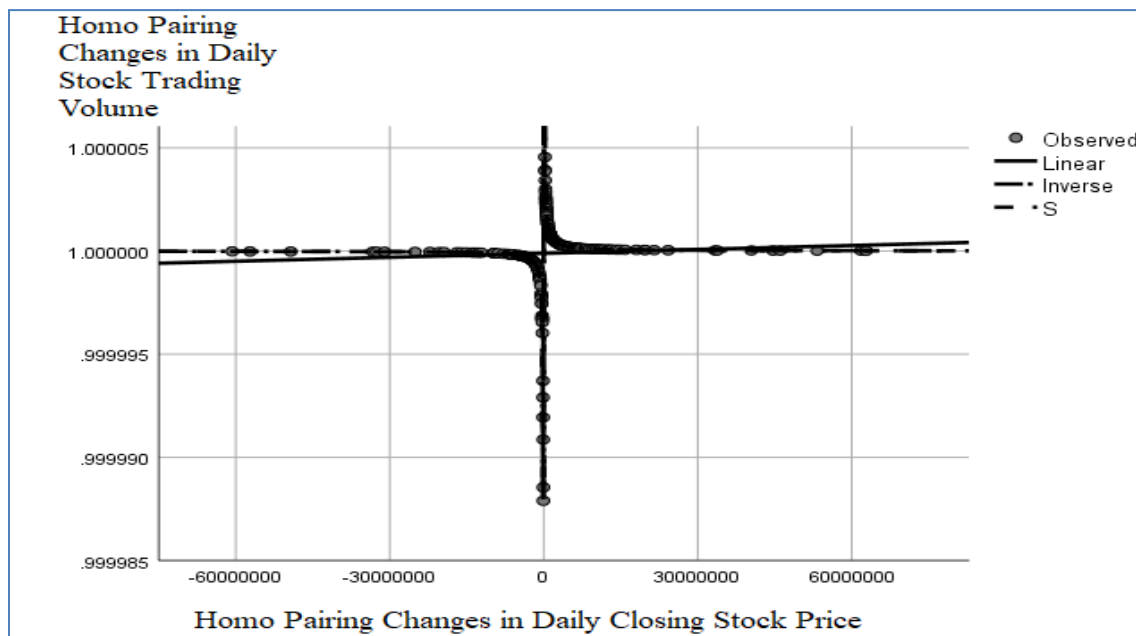


Figure 9: Curve estimation of homo pairing changes in stock price and stock trading volume
Source: Author (2022).

The figure exhibits two classes that are described by the non-linear pairing functions- the inverse and S-curved models. Each model is described by two non-pairing classes, the PPP and NPP classes. Therefore, since the inverse model has a lambda coefficient is 1, and the constant value is 1 the NYSE market technical model is described as;

$$\partial_{VP} = 1 + \frac{1}{\partial_{PV}} ; \text{ PPP class model}$$

$$\partial_{VP} = 1 - \frac{1}{\partial_{PV}} ; \text{ NPP class model}$$

On the other hand, the NYSE market technical model can be determined by the S-curved function. Since the S-curved model has a lambda coefficient of 1 and the constant value is 1.451E-12, therefore, the NYSE market technical model is described as;

$$\ln \partial_{VP} = 1.451 \times 10^{-12} + \frac{1}{\partial_{PV}} ; \text{ PPP class model}$$

$$\ln \partial_{VP} = 1.451 \times 10^{-12} - \frac{1}{\partial_{PV}} ; \text{ NPP class model}$$

In conclusion, both empirical models- inverse and S-curved models that represent the NYSE market technical models, proved that the homo pairing changes in the daily trading volume relative to the change in the daily closing stock price are non-linearly related to the homo pairing changes in the daily closing stock price.

4.7 Fundamental analysis in the NYSE market

The fundamental analysis of the NYSE market was done by regressing the homo-pairing changes in the daily closing stock price relative to the change in daily stock information, ∂_{PD} , and homo pairing changes in daily stock information relative to the change in daily closing stock price, ∂_{DP} . The study hypothesis (alternative hypothesis) is that the homo pairing change in daily closing stock price relative to the change in daily stock information has a non-linear pairing impact on the homo pairing changes in daily stock information relative to the change in the daily closing stock price in the NYSE market. The HHP regression model was run using SPSS software and the model summary and parameter estimates were provided (Table 9).

Table 9: Model Summary and Parameter Estimates in the NYSE market

Dependent Variable: Relative-Days homo pairing changes of daily closing stock price (∂_{PD})							
Model Summary						Parameter Estimates	
Equation	Ψ^2	F	df1	df2	Sig.	Constant (λ_0)	λ_1
Linear	0.014	3.322	1	237	0.070	0.994	3.513E-5
Inverse	0.939	3646.785	1	237	0.000	1.000	0.639
S	0.984	14448.241	1	237	0.000	7.334E-5	0.785

The independent variable is the Relative price- homo pairing changes of daily stock information(∂_{DP})

Source: Author (2022).

Table 9 shows the HHP regression model summary and its parameter estimates. The table provides the three parameters of the three models; the linear, inverse and S-curved models. The linear model has a positive lambda coefficient of 3.513E-5 and a constant value of 0.994, a p-value of 0.070 which is greater than the recommended critical value of a significant level of 0.05. The model is poorly determined as it has a linear pairing determination coefficient, Ψ^2 of 0.014. Therefore, we reject the null hypothesis and accept the alternative hypothesis at a 95 percent of confidence level since its p-value of 0.070 is greater than the critical value of 0.05. The second model is the inverse model which has a positive lambda coefficient of 0.639, a non-linear pairing determination coefficient, Ψ^2 of 0.939 and a constant value of 1.000, with a p-value of 0.000 which is less than the recommended critical value of the significant level of 0.05. Hence, we reject the null hypothesis and accept the alternative hypothesis at a 95 percent of confidence level since its p-value of 0.000 is less than the recommended critical value of 0.05. The third model is the S-curved model which has a positive lambda coefficient of 0.785, a constant value of 7.334E-5, a non-linear pairing determination coefficient, Ψ^2 of 0.984 and the p-values of 0.000, which is less than the recommended critical value of the significant level of 0.05. Hence, we reject the null hypothesis and accept the alternative hypothesis at a 95 percent of confidence level.

The two significant models- inverse and S-curve models are described in non-linear pairing classes; the PPP class and NPP class. The classes are explained by the homo-hetero scattering plots established by curve estimation (Figure 10). The curve estimation plots the scattering of homo pairing changes in daily closing stock price relative to the changes in the daily stock information and the homo pairing changes in daily stock information relative to the change in the daily closing stock prices in the NYSE market. The classes are described by the non-linear pairing function-the inverse and S-curve models.

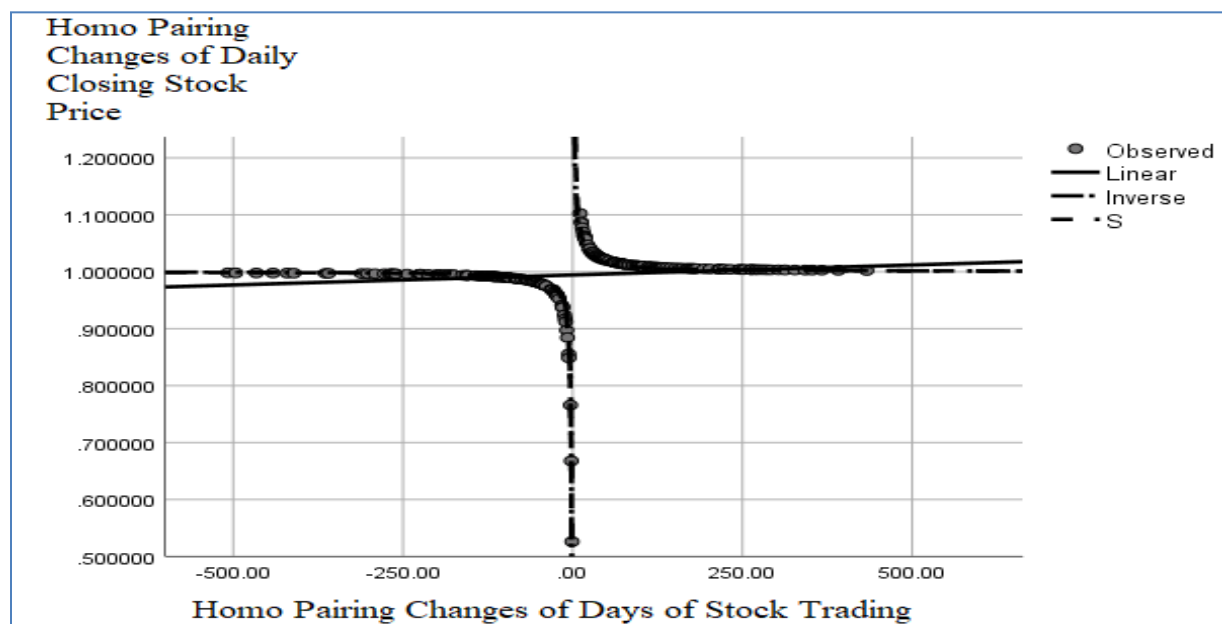


Figure 10: Curve estimation of homo pairing changes in stock price and daily stock information

Source: Author (2022).

Figure 10 shows the curve estimation of the homo pairing changes in the daily closing stock price relative to the daily stock information and the homo pairing change in daily stock information relative to the change in the daily closing stock price in the NYSE market. The figure exhibits two classes that are described by the non-linear pairing functions- the inverse and S-curved models. Each model is described by two non-pairing classes, the PPP and NPP classes. Therefore, since the inverse model has a lambda coefficient is 0.639, and the constant value is 1 the NYSE market fundamental model is described as;

$$\partial_{PD} = 1 + \frac{0.639}{\partial_{DP}} ; \text{ PPP class model}$$

$$\partial_{PD} = 1 - \frac{0.639}{\partial_{DP}} ; \text{ NPP class model}$$

On the other hand, the NYSE market fundamental model can be determined by the S-curved function. Since the S-curved model has a lambda coefficient of 0.785 and the constant value is 7.334E-5, therefore, the NYSE market fundamental model is described as;

$$\ln \partial_{PD} = 7.334 \times 10^{-5} + \frac{0.785}{\partial_{DP}} ; \text{ PPP class model}$$

$$\ln \partial_{PD} = 7.334 \times 10^{-5} - \frac{0.785}{\partial_{DP}} ; \text{ NPP class model}$$

In conclusion, both empirical models- inverse and S-curved models that represent the NYSE market fundamental models proved that the homo pairing changes in daily closing stock price relative to the changes in the daily stock information are non-linearly related to the homo pairing changes in daily stock information relative to the daily closing stock price.

4.8 Discussion

The study aimed to examine the technical and fundamental hypotheses in NYSE, NASDAQ and S&P 500 stock exchange markets by application of the homo-hetero regression model. The main determinants (variables) that were examined are stock trading volumes, stock closing prices and stock information available in the stock exchange market about the specific stock. The study evidenced that both the technical and fundamental features of the markets are defined by the inverse and S-curved relations. The inverse and S-curved relationships are evidenced in all stock exchange markets. Furthermore, the study evidenced that the homo pairing change in the daily trading volume relative to the change in the daily closing stock price has an inverse and S-curved relationship with the homo pairing changes in the daily closing stock price relative to the change in the daily trading volume. Moreover, the homo pairing change in the daily closing stock price relative to the change in the daily stock information has an inverse and S-curved impact with the homo pairing change in the daily stock information relative to the change in daily closing stock price. The inverse and S-curved models evidenced in the study are described in two distinctive pairing classes, namely positive-positive pairing (PPP) class and negative-positive pairing (NPP) in both fundamental and technical analysis. In the technical analysis, the study evidenced the

empirical relationship /model of $\partial_{VP} = 1 \pm \frac{1}{\partial_{PV}}$. This means that the ∂_{VP} in the NYSE, NASDAQ and S&P 500 stock exchange markets vary inversely to the ∂_{PV} . In other words, we conclude that the homo pairing changes in the daily stock trading volumes relative to the changes in daily closing stock price are reciprocally related to the homo pairing changes in the daily closing stock price relative to the changes in the daily trading volume. This can be interpreted that, a unit homo pairing change of daily closing stock price relative to changes in the daily trading volume resulting in two effects in the markets. Some investors will lose (getting zero returns), but other investors will gain (2), that $\partial_{VP} = 1 \pm 1$, when $\partial_{PV} = 1$, then $\partial_{VP} = 0$ or 2. This phenomenon evidenced the nature of volatility of the market, that there is no assurance of the investor to gain or lose when the unit homo pairing changes in daily closing stock price relative to the changes in daily trading volume happened. This is why the stock prediction becomes harder and more complex as suggested by some scholars (Karin et al., 2021; Gharehchopogh et al., 2013; Bhuriya et al., 2017; Teachman, 1995; Srivinary et al., 2022). This can be explained that when a unit homo pairing changes happened in daily closing stock price the investors can expect to profit (gain) or loss. This means some investors will not have relevant information about the stock price; hence they will underprice or overprice the stock's valuation. From this aspect, we confirmed that the technical hypothesis does not fully capture the investor buying or selling (trading) decisions in the stock markets. It does not offer reliable predictive information; it can forecast both the gain and loss at the same time. This finding confirms the studies of Wang (2014), Mpofu (2012), and Alhussayen(2022).

On the other hand, the study examined the fundamental hypothesis in the NYSE, NASDAQ, and S&P 500 stock exchange markets. The empirical study found that the fundamental analysis in the markets is described by the general inverse model

$$\partial_{PD} = 1.008 \pm \frac{0.909}{\partial_{DP}} \text{ in S\&P 500,}$$

$$\partial_{PD} = 1.001 \pm \frac{0.986}{\partial_{DP}} \text{ in NASDAQ and}$$

$$\partial_{PD} = 1 \pm \frac{0.639}{\partial_{DP}} \text{ in NYSE.}$$

The general meaning is that the relationship between the homo pairing change in daily closing stock price relative to the changes in daily stock information, ∂_{PD} , and homo pairing changes in the daily stock information relative to the change in the daily closing stock price, ∂_{DP} , is described by the inverse model. The unit change of the ∂_{DP} , i.e., $\partial_{DP} = 1$, the fundamental empirical equation becomes, $\partial_{PD} = 1.008 \pm 0.909$ in S&P 500, $\partial_{PD} = 1.001 \pm 0.986$ in NASDAQ and $\partial_{PD} = 1 \pm 0.639$ in NYSE. This means, that investors relying on the daily stock information (fundamentalist) reduced the vulnerability or risks of the stock underpricing or overpricing his financial asset (stock). For example investor in S&P500 who used the fundamental hypothesis to predict the impact of the unit change of stock information on the price of the stock the next day, he will have a pay-off of the range of 1.008 ± 0.909 ; that is, the stock price predicted to raise by a ratio of 1.909 (higher) or 0.099 (lower). Therefore, the fundamental analysis is more reliable than the technical analysis. Moreover, the investor in NYSE used the fundamental analysis/model, his returns (pay-off) have a range of 1 ± 0.639 , that is he expected

his return or price to raise by a ratio from 1.639 (higher) to 0.361 (lower) for each change of the day of stock information in the market. In using the fundamental analysis an investor can expect or predict to get higher or lower returns (pay-off). This approach is the best as an investor can expect no loss from the investment. The approach is more reliable than the technical analysis which predicts loss and gains only. This finding goes in line with Huang, et al., 2021, Mahajan and Singh (2008), and Wu et al., (2022).

In addition, the study confirms the non-linear and volatility of the stock exchange market since we found both technical analysis and fundamental analysis are described with gain and loss (PPP class and NPP class). One question to ask is why the three markets are defined with the same technical models and slightly differ in fundamental models. Purposely, the study collected data from the three largest stock exchange markets in the world to have a comparison of facts/findings. Generally, the findings are the same for each stock exchange. Do we conclude that technical and fundamental analysis in all the stock markets in the world can be defined by inverse and S-curved models? The answer is no! Bajzik(2020) contented that one cannot rely on the general conclusion on the stock exchange market, this is because the predictability of the stock price(returns) varies with different markets and the nature/type of the stock. Now we conclude the S&P500, NASDAQ and NYSE markets have similar or share a lot of empirical characteristics of the technical and fundamental determinants/factors. However, we call up more research in this area by using this HHP regression model in other stock exchange markets to enhance the empirical cross-validity of the general conclusion of this study.

Specifically, the use of the HHP regression in analysing both the technical and fundamental hypothesis at the stock exchange market is valued as the model can able to detect/capture both linear and non-linearity behaviour of the determinants of the stock exchange markets. The use of traditional linear regression could not detect the non-linearity (Cai and Gao, 2017; Sahoo and Charlapally, 2015; Chaudhary et al. 2018; Gupta and Wagalakshmi, 2019).

5. Conclusion and recommendation

The paper aimed to examine the impact of homo pairing changes in daily closing stock price relative to the change in daily stock information on homo pairing changes in the daily stock information relative to change in daily closing stock prices (fundamental analysis); and the impact of homo pairing changes in the daily trading volume relative to daily closing stock price and the homo pairing changes in daily closing stock price relative to the daily trading volume (Technical analysis) in S&P 500, NASDAQ and NYSE markets. The study applied the Homo-Hetero Pairing Regression Model, simply the HHP regression model. This model was developed to identify the linear and non-linear behaviour of data, hence being an appropriate econometric predictive model for homo paired data such as stock prices. The study found that both the technical and fundamental hypotheses are defined by the inverse and S-curved models in the three stock exchange markets. This means that the non-linearity relationship between the changes in stock information and closing stock price (fundamental hypothesis) and the changes in closing stock price and stock trading volume (technical hypothesis) is confirmed. Moreover, the study confirmed that the fundamental analysis is more reliable that can predict accurately more than technical analysis. Therefore the study concluded that the optimal prediction of the stock price or return is achieved by the fundamentalists in the stock exchange markets. However,

the expected returns range from higher to lower gains (profits). The study recommends that the stock investors prioritise the use of fundamental hypothesis /analysis to make their stock investment decision. In other words, the study recommends that stock investors be fundamentalists.

On the other hand, the study found that the homo-hetero pairing regression model is the best analysis to study the homo-paired data such as the stock prices. This is because the models capture the non-linearity and linearity behaviour of the homo-paired data such as stock price. Hence the study recommends the application of this regression model to the study of financial markets, economics, psychology, and medicine. Specific recommendations for the application of the HHP regression model are the prediction of water waves in the investigation of hydrodynamic and erosion-accretion processes on shallow depths.

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